Shape Dependent Form of Newton's Third Law of Motion

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Submitted on 20-02-2022

Abstract

Newton's third law (law of conservation of momentum) is being used in all branches of physics right from Newtonian Mechanics to the ideal rocket equation. The law was originated in Principia without mathematical equations. Action and reaction are expressed in terms of push, pull, or force (weight, mg).

In simple mathematical descriptions of freely falling and rebounding bodies (and in various types of collisions), the law neglects the shapes of bodies (spherical, umbrella-shaped, polygon, cone, thin pipe, flat, typical shape, etc.) and other related factors. These experiments need to be understood quantitatively. Some equations based on elastic collisions (using conservation of momentum and kinetic energy) lead to inconsistent results regarding the Special Theory of Relativity.

The elusive factors can be taken into account by generalizing the law as: Reaction $= -Q \times$ Action $= -$Action $\times (Q_{\text{shape}} \times Q_{\text{composition}} \times Q_{\text{target}} \times Q_{\text{other}})$ where Q_i accounts for effects of shape, composition, target, and other factors. Various experiments have been suggested to calculate Q. If $Q = 1$, then the generalized form reduces to Action $= -$Reaction and is more practical. Likewise, Tsiolkovsky's and other related equations vary.

The rocket motion is controlled by a computer algorithm, whereas the motion of fireworks is understood by Tsiolkovsky's equation.

Key words: Third law, Shape, characteristics, rebounding bodies.

1 Introduction

Newton defined the third axiom (a rule or principle that most people believe to be true) or the law of motion in the Principia [1] on page 20 as: "*To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and* *directed to contrary parts.*"

$$Action = -Reaction \tag{1}$$

While explaining the third law, Newton further quoted its wide range of applicability as:
"*Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone.*"
Thus, Newton justified the equality of action and reaction (but opposite in direction) in the above two examples. Newton expressed the action and reaction in terms of push or pull, which is a force. The force exerted by one body (Action) = - force exerted by the second body (Reaction) The law is obeyed if the magnitude of the reaction is precisely equal to the action, and their direction is precisely opposite (180^o with the original direction). Thus, the simultaneous obedience of both conditions (equality of magnitudes and opposite directions) is the fundamental requirement of the consistency of Newton's third law. In these two examples, Newton considered cases when forces (push or pull) act on stones, but the stones (bodies) remain at rest. In the third example, Newton considered a scenario where one body impinges on another, leading to the conservation of momentum. In this example, Newton considered moving bodies due to interactions, making various dynamical cases feasible.

2 Universal and Absolute Applicability of the Definition and Equation of the Third Law

Newton used the term "body" in the third law, which he had defined as a mass or quantity of matter in Definition I of Principia. The Definition I of Principia states: "Quantity of Matter" (synonymous with mass or body). The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.

"*It is this quantity that I mean hereafter everywhere under the name of body or mass.*" Examples of Quantity of Matter or mass or body in other Definitions in the Principia [1] include: iron, loadstone, sling, whirling stone, projectile, earth, moon, heavy mountains, bodies (whether heavy or light, great or small), solids, whole body, parts of the body, ship, sailor, pendulum clock, Jupiter, stars, vessels, long cord, etc. These are macroscopic bodies. Newton's third law of motion applies to all bodies at the macroscopic level (as Newton himself explained in the Principia). However, in limiting cases, the law is equally applicable to point or particle dynamics as well. Newton's third law is taught qualitatively even at the elementary level for observable bodies, such as balls, walls, bullets, guns, boats, swimmers, persons, and books. However, for completeness, the law must be quantitatively confirmed in all cases (macroscopic or microscopic) as it is applicable to all types of bodies. This is the theme of the discussion. While qualitative study may be philosophical, quantitative study is scientific.

2.1 Diverse characteristics of bodies in nature

The definition and equation 1 of the third law are general and universally applicable to all perceivable bodies (projectiles and targets) of different shapes, characteristics, and compositions.

2.1.1 Various Bodies

Various examples of such bodies include wool, wood, cloth, springs, steel, rubber, clay, kneaded flour, chewing gum, sponges, plastics, porous materials, air/fluid-filled artifacts, super balls, sky balls, and many others. Projectiles and targets (bodies) can also be in liquid or semi-liquid forms, and air-filled balls are considered bodies as well. Thus, Newton's law applies to bodies in diverse forms, characteristics, shapes, and structures. However, in the Principia, Newton considered only the simplest possible examples.

2.1.2 Characteristics of the Bodies

Different bodies can act as projectiles or targets unconditionally, exhibiting various characteristics such as inherent composition, elasticity, bounciness, plasticity, flexibility, rigidity, magnitude, size, and distinctiveness of interacting bodies or modes of interaction. The super ball and sky ball are notable examples of extremely elastic bodies, discovered in the 1960s and 1970s, which bounce more times than ordinary rubber balls [2]. The physical reasons for their elastic behavior differ from those of other materials. For instance, the cis-poly-isoprene chain in rubber forms a coiled structure that can be stretched like a spring. In contrast, the super ball contains synthetic polymer polybutadiene, hydrated silica, zinc oxide, stearic acid, and other ingredients, making it an extremely elastic and bouncy ball. On the other hand, iron (a metal) has entirely different characteristics and a body-centered cubic (bcc) crystalline structure.

2.1.3 Shapes of Bodies

Shapes of bodies can vary greatly, including spherical, semi-spherical, umbrella-shaped, triangular, square, hexagonal, polygon, cone, long, thin pipe, flat, irregular, or any feasible typical shape. Asymmetry, orientations (angle of fall), etc. imply that Newton meant his law for all bodies (projectile and target) indiscriminately. So, the law must be experimentally verified for all interacting bodies quantitatively, i.e., both in magnitude and direction. Thus, the third law of motion as given in Principia is absolute and universal, as there is no condition in the definition and applications.

3 Freely Falling Bodies

In freely falling bodies, action (force or weight) is the same if the mass is the same. A basketball falls due to gravity and rebounds, obeying Newton's third law, i.e., equality of action and reaction (but the direction is opposite). Action and reaction occur in pairs on different bodies [3]. Also, various balls or bodies rebound when hit horizontally on the ball. However, these are qualitative observations. Quantitative observations are scientific.

In freely falling bodies in a vacuum, action or force or weight (mg) of the body is precisely known. Let a spherical ball (rubber/plastic, super ball, and sky ball) or any other body of mass 1 kg be fabricated and freely fall in a vacuum from a height of 1 m (H). The ball or body is attracted by the gravitational force of the Earth, i.e., weight (mg or 9.8 Newton), which is action or action force. Thus, action is caused by gravitational force, and acceleration due to gravity (equal to g) is experienced by the body. The time taken by the body to reach the bottom is given by

$$H = \frac{1}{2}gt^2 \tag{2}$$

,is 0.45 s. According to Newton's third law: Action = Force = Weight, i.e.

$$W = mg = 1kg \times 9.8m/s^2 = 9.8Newton \tag{3}$$

When the ball strikes the floor, then, due to mutual interactions between the ball and the floor, reaction arises spontaneously, i.e., mutual simultaneous interactions are implied in the third law [3]. Action and reaction occur in pairs on two different bodies, here one body is a ball, and the other is a surface. Now, according to the third law, reaction or reaction force would be precisely the same as action, i.e., 9.8 newtons (but opposite in direction), for a rubber ball, super ball, a sky ball, or any other body (having mass 1 kg). So, bodies would rebound upward exactly in the opposite direction (indicated by a negative sign) to the original height of 1 m. Thus:

$$Reaction = -Action = -9.8Newtons \tag{4}$$

As the body falls on the surface, it exerts an action force on the surface. The surface also exerts a reaction force on the body, thus pushing the body upward (opposite in direction). The higher the reaction force (upward force; action is downward), the higher the upward distance traveled by the ball (H_{reb}). The height to which the body rebounds (H_{reb}) is a measure of reaction. As the reaction and action are equal and opposite (according to Newton's law), the height to which the body rebounds (H_{reb}) must be equal to the height from which the body is dropped (H).

$$H = H_{reb} \tag{5}$$

For the precise validity of Newton's third law, the body must rebound to its original height at the same time, exactly retracing the original path. This prediction may be justified for a particular body (shape, composition, etc.) and target quantitatively. Similarly, equations for horizontal motion can be studied, reducing the sources of errors or uncertainties as far as possible. There are more equations, such as eqs. (6-10), which directly follow Principia's third application of Newton's third law. Qualitative observations may be philosophical, but quantitative observations are scientific.

3.1 Quantitative Discussion of Free Fall and Rebounding Bodies of Different Possible Shapes

Bodies of mass 1 kg may have different shapes (umbrella-shaped, square, hexagonal, cone, thin pipe, flat, irregular, or typical shape); thus, mass does not vary with shape. So, according to the definition of the third law, the action (hence reaction) does not vary with shape if mass remains the same. Therefore, action and reaction are both independent of shape, as in eqs. (3) & (4) , and rebounding distances should not depend on shape. The composition and mass of these artifacts are precisely the same, i.e., rubber (plastic) or a suitable material and mass (1 kg), as in the case of a spherical body. Thus, the inherent characteristics of bodies remain the same, as mass and composition remain unchanged; only the shapes are varied for experiments. Consequently, action and reaction remain the same, i.e., 9.8 Newton. Hence, the definition of the third law requires that bodies must rebound to the same height.

3.2 Repeated Qualitative Observations at the Macroscopic Level

There are qualitative observations at the macroscopic level that bodies of different shapes (same mass and composition) rebound to lesser heights compared to spheres when all strike the same surface. However, action and reaction are the same in all cases, as in eqs. (3) & (4). These observations need to be confirmed quantitatively, which is the theme of discussion. A typical flat body does not rebound at all or rebounds to just a few centimeters; however, it has the same action and reaction (9.8 newtons) as a spherical body. As the reaction is the same, bodies must rebound to precisely the same height in the exactly opposite direction, i.e., H_{reb} and H must be the same in all cases. It is observed that unsymmetrical bodies (semi-spherical, cone, triangle, long thin pipe, irregular shape, etc.) of the same mass and material rebound from the same floor at different angles. They do not rebound exactly in the opposite direction like spherical bodies, but rebound at different angles, which is not consistent with the directional part of the law. The rebounding of such bodies (different shapes) is not consistent with the third law, both in magnitude and direction.

3.2.1 Conceptual Limitations at the Time of Newton

Newton initiated physics, separating it from natural philosophy, and explained phenomena philosophically and geometrically in the Principia [1]. Newton did not provide mathematical equations, as mentioned by Bernard Cohen in his celebrated critical and explanatory treatise on The Principia [4]. Newton did not discuss these cases due to conceptual limitations at that time. The acceleration due to gravity (g) was determined (as 9.80665 m/s^2) for the first time in 1888 in France, i.e., 202 years after the enunciation of the law of gravitation by Newton. This value was adopted at the third General Conference on Weights and Measures, held in 1901, thus defining weight. Thus, weight was defined 215 years after the enunciation of Newton's law of gravitation in 1686. These equations and scientific perceptions did not exist at the time of Newton. However, for the quantitative validity of the law, it must be studied completely in specific observations at both macroscopic and microscopic levels.

3.2.2 Loss of Energy in Interactions

Further energy may be lost in various forms, such as heat energy (mass x specific heat x rise in temperature), sound energy, etc. These terms and other factors are not taken into account in the definition and eqs. (1, 3 & 4). However, Newton's law is in terms of force, but now dissipation of energy is considered. It only stresses the equality of action force and reaction force. The loss of energy due to heat requires measurement of the rising temperature of the body (which is practically negligible) in the relevant equation (m $\times$ s $\times$ δt). Now, in any case, quantitative precision of the highest level is possible. Let the amount of energy dissipated during collision be $E_{dissipated}$. The dissipated energy must be capable of explaining the flat body rebounding to a lower height (H - H_{reb}), i.e., 1 cm or different, for the validity of the law if the law is obeyed.

3.2.3 Area of Contact and Symmetrical/Asymmetrical Interacting Bodies (Projectile and Target)

The area of contact (AOC) may be understood as the actual area of the body that comes into direct contact with the surface (of the floor) when a reaction arises due to mutual interactions (of body and floor). The area of contact may be the same for various bodies if these are properly fabricated. This aspect can be understood by fabricating bodies such as cones (pointed base), long pipes, triangles, typical bodies, etc., such that the area of contact (AOC) of each body is the same as that of the sphere. The triangle may be dropped at the pointed edge. Experiments with a thin, long pipe shape (rubber pipe having the same mass as that of the sphere, same AOC) and the sphere would be significant. In such experiments, even at a qualitative basis, unsymmetrical bodies (semi-spherical, cone, triangle, irregular shape, etc., 1 kg, say) would not rebound in exactly the opposite direction. Thus, the directional part of the third law of motion is not obeyed experimentally. However, action and reaction would be the same as in eqs. (3-4), as the masses of bodies are the same in free fall. In this case, experimentally, the orientation (the angle at which bodies fall) and symmetry (distribution of mass of the body due to shape, size, etc.) appear to be equally significant factors. These factors are required to be specifically studied in experiments.

Thus, Newton's law must not be regarded as quantitatively true without experiments, as in such experiments, shape, composition, nature of the target, symmetry, etc., play significant roles. All these factors must be quantitatively and specifically measured [5-14]. Due to this reason (as Newton's law does not take all factors into account), the law is generalized as in eqs. (18- 20) in sections (5.0-5.1). For precise validity of the law, quantitative and specific experiments are absolutely necessary.

4 The Third Application of Newton's Third Law in the Principia or the Law of Conservation of Momentum

The third application [1] of the law in the Principia, on page 20, describes the situation when one body impinges on another. If a body impinges upon another and, by its force, changes the motion of the other, that body also (because of the equality of the mutual pressure) will undergo an equal change in its own motion, towards the contrary part. Consider that body A (projectile) of mass m applies force and changes the motion of body B (target) of mass M. Let the initial velocity of body A be $U_{initial}$ and that of body B be $V_{initial}$. After the collision, the velocity of body A becomes U_{final} and that of body B becomes V_{final}. Change in motion of body B when body A impinges on it = -change in motion of body A

$$MV_{final} - MV_{initial} = -(mU_{final} - mU_{initial})$$
$$or$$
$$mU_{initial} + MV_{initial} = mU_{final} + MV_{final} \quad (6)$$

which is the law of conservation of momentum. At the macroscopic level, in different ways, eq. (6) may be experimentally confirmed quantitatively. Further,

$$U_{final} = U_{initial} + (V_{initial} - V_{final})M/m \quad (7)$$

$$V_{final} = V_{initial} + (U_{initial} - U_{final})m/M \quad (8)$$

If the mass of the projectile is equal to the mass of the target (m=M), irrespective of other parameters, then

$$U_{initial} + V_{initial} = V_{final} + U_{final} \quad (9)$$

As there are no constraints (about mass, shape, size, composition, surfaces, etc.) on eq. (6), it is absolutely and conceptually applicable in all cases involving various parameters. Thus, experimental confirmation of equations is needed over a wider range of parameters. The equation for the law of conservation of momentum (Newton's third law of motion) for the bullet-gun system is given by:

$$M_{gun}V_{gun}(\text{backward momentum of gun})$$
$$= -m_{bullet}v_{bullet}(\text{forward momentum of bullet})$$
$$or$$
$$V_{gun} = -m_{bullet}v_{bullet}/M_{gun} \quad (10)$$

where all terms have their usual meanings. Equations (6- 10) need to be experimentally confirmed quantitatively under different conditions.

Further, Tsiolkovsky [15] obtained the forward velocity of a rocket (V), using the law of conservation of momentum (third law):

$$\text{Forward momentum of rocket}(Mdv)$$
$$= \text{–Backward momentum of the exhaust}$$
$$\text{(fire, gases, smoke, sparks, etc.) of rocket}(dMVe) \quad (11)$$

or

$$V = V_e ln(\frac{M_0}{M}) \quad (12)$$

where V is the velocity of the rocket at any time, V_e is the backward escape velocity of exhaust, M_0 is the original mass (t=0) of the rocket, and M is the mass at time t. This equation is known as the ideal rocket equation and describes the motion of fireworks.

Goddard derived the equation of rocket (for vertical motion) taking into account the effect of gravity. Now, this equation is expressed in terms of specific impulse (I_sp) and standard gravity (g_0) as:

$$V = V_e ln(\frac{M_0}{M}) = g_0 I_{sp} ln(\frac{M_0}{M}) \quad (13)$$

Historically, rockets were first successfully demonstrated in 1232 in the Chinese-Mongol battle of Kai-fung-fu fireworks (which were not controlled externally by a computer algorithm). These rockets were completely uncontrolled once fired, like fireworks today. Newton did not mention rocket motion in Principia (1686). Tsiolkovsky [15] used the third law of motion in the form of conservation of momentum to derive the rocket equation in 1897 (211 years after the enunciation of the third law) and published it in 1903.

4.1 The Equations of One-Dimensional Elastic Collision Involve the Third Law of Motion

In a one-dimensional elastic collision, the conservation of linear momentum (a form of the third law of motion) and kinetic energy are used simultaneously. The final velocity of the projectile (U_{final}) and the final velocity of the target (V_{final}) are given by standard textbooks [3]:

$$U_{final} = \frac{([m-M]U_{initial} + 2MV_{initial})}{(m+M)} \quad (14)$$

$$V_{final} = \frac{([M-m]V_{initial} + 2mU_{initial})}{(M+m)} \quad (15)$$

Further,

$$U_{initial} = \frac{(U_{final}[m+M] - 2MV_{initial})}{(m-M)} \quad (16)$$

$$V_{initial} = \frac{(V_{final}[m+M] - 2mU_{initial})}{(M-m)} \quad (17)$$

These equations need to be experimentally confirmed quantitatively at the macroscopic level, specifically for bodies of different masses, shapes, sizes, compositions, surfaces, etc.

4.2 Theoretical Invalidity of Eqs. (16- 17)

Eqs. (16- 17) are not valid under certain conditions, as discussed below.:

1. If the numerator in eq. (16), i.e., $[U_{final}(m+M)–2MV_{initial}]$, is non-zero, and m-M is sufficiently smaller (masses of both projectile and target are nearly equal). Thus, under these conditions, the velocity $U_{initial}$ may be equal to the speed of light (c), or unrealistic results are obtained. This is not consistent with the Special Theory of Relativity.

2. Similar results follow from eq. (17) if M-m is sufficiently smaller and the numerator is non-zero.

3. Further, when m = M, and the numerator is non-zero, then velocities in eqs. (16- 17) may become infinity (∞) theoretically. This is again not justifiable.

This impartial interpretation gives the strongest evidence that the above equations must be experimentally checked quantitatively.

5 Speculative or Theoretical Form of the Third Law of Motion

In one way or another, some deviations from Newton's third law of motion are discussed in standard literature, such as the European Journal of Physics in the article "Breaking Newton's Third Law: Electromagnetic Instances"

[16]. This discussion involves moving charged particles. Newton's third law can be violated in certain non-equilibrium (out-of-balance) situations for mesoscopic particles in statistical mechanics [17]. Newton's third law of motion is not strictly correct with interactions between two bodies separated by a large distance; such deviations are found when electric and magnetic fields [18]. Roger Shawyer proposed the movement of a rocket (prototype) with the help of microwaves without exhaust in EM Drive. The proposal was not accepted by scientists, as it violates the third law of motion or conservation of momentum. However, NASA scientists confirmed the above perception, i.e., the system was consistently performing with a thrust-to-power ratio of 1.2±0.1mN/kW, and enumerated many potential sources of error [19]. However, it needs to be supported by repeated experiments [20]. Needless to mention, Newton's Corpuscular theory of light has been replaced by Huygens's wave theory of light. Also, Newton's equation for the speed of sound in media ($v=\sqrt{\frac{P}{D}}$) has been modified by Frenchman Laplace ($v = \sqrt{\gamma\frac{P}{D}}$) so that theoretical results may coincide with experimental findings. P is pressure, D is the density of the medium, and γ is the ratio of two specific heats. Thus, improvements in the law are feasible as experimental and theoretical situations improve. Also, some very genuine experiments, as discussed in previous sections, have not been conducted quantitatively even at the macroscopic level. These specific experiments are absolutely necessary for a complete understanding of the law. Theoretical generalization of the law has been anticipated to account for all factors that are elusive to Newton's third law of motion.

"*To every action, there is always a proportional reaction, depending upon the shape, characteristics of bodies, etc. of the process.*" Mathematically,

$$\begin{aligned} Reaction &\propto Action \\ &or \\ Reaction &= -Q \times Action \end{aligned} \tag{18}$$

where Q is the coefficient of proportionality (dimensionless). Q accounts for shapes, sizes, characteristics, compositions of interacting bodies, the nature of the surface on which interactions take place, external factors, and all elusive factors that are not accounted for by the original law (Reaction = - Action). The effects of various factors are determined experimentally; the case-to-case study of Q is exceptionally complex and has not been purposely done. So, the law initially needs to be considered (or confirmed) for standard conditions for simplicity, then its study may be extended to non-standard conditions. For mathematical simplicity, the value of Q may be expressed empirically.

5.1 Empirical Model of Coefficient Q in Eq.(18)

Q engrosses in itself the effects of shape, size (via coefficient Q_{shape}), effects of nature, characteristics, and composition ($Q_{composition}$), effects of nature, the magnitude of target (Q_{target}), and effects of all other involved factors (Q_{other}). As mentioned above, Q depends on various factors, so it may be expressed in one of the simplest ways empirically as below:

$$Q = Q_{shape} \times Q_{composition} \times Q_{target} \times Q_{other} \quad (19)$$

$$Reaction = -Action \; (Q_{shape} \times Q_{composition} \times Q_{target} \times Q_{other}) \quad (20)$$

If each coefficient is unity or the net effect is unity, then eq. (18) or eq. (20) is reduced to eq. (1, 3-4). In general, gas laws (Boyle's, Charles', Gay-Lussac), Hooke's laws, laws of friction, etc. are empirical laws and based on observations. The coefficients are determined experimentally.

Also, Newton [21] has described the law of gravitation (without any equation or derivation) in Vol II, Book III of the Principia in Propositions I-IX in an empirical way in expressions. Also, Bethe and Weizsäcker had given a semi-empirical mass formula for the liquid drop model of the nucleus.

6 The Perception of Standard Bodies and Various Coefficients

The standard bodies (projectile and target) have been considered to simplify the understanding of complex and weird applications of the third law. Then phenomena may be studied step by step quantitatively. If the value of Q is unity or the net effect of Qi's is unity, then eqs. 18-20 become eq. (1). For simplicity, the body may be standard if it is spherical, mass 1 kg (say), is made of suitable material, and rebounds to precisely the original height ($H_{reb} = H$) exactly in the opposite direction after striking the suitable target. Further, the composition of the body (maybe plastic or rubber, super ball, sky ball, or a suitable combination) is required to be judiciously chosen. Practically, numerous pairs of projectiles and targets are possible. Further, the target may be standard if it is a sheet of 0.50 m thick, 3 m x 3 m in length and breadth of uniform plastic or suitable material.

If $Q_{composition}$, Q_{target}, Q_{other} are unity (standard conditions) but the body differs in shape (from standard spherical), and the body rebounds (H_{reb}) to a different height than H. In case the value of H_{reb} is observed $\frac{1}{2}$ m (say), then the value of Q_{shape} would be $\frac{1}{2}$. Then eq. (18) or eq. (20) will vary due to shape only as,

$$Reaction = -\frac{1}{2} Action (Q = Q_{shape} = \frac{1}{2}) \quad (21)$$

Then this effect would be due to shape only, and accordingly, the value of Q would be determined. The value of the area of contact (AOC) may be regarded as associated with Q_{shape} as

$$Q_{\text{shape}} \propto \frac{1}{area\ of\ contact} \quad or \quad Q_{\text{shape}} = \frac{k}{area\ of\ contact} \quad (22)$$

k depends upon characteristics of bodies and involved experimental conditions. The value of k would depend upon case-to-case study due to weird factors. Thus, for simplicity, bodies may be divided into different sub-categories (depending upon structural properties) due to the diversity in nature of experiments. Anybody may act as a projectile in some cases and a target in some other cases. For a projectile, the standard target may be one for which action is precisely equal to reaction. In such cases, the most peculiar observation would be if the target consists of material of super balls or sky balls freely falling on this target. In some cases, the nature of the target varies due to energy imparted to it by the projectile, e.g., when the ball falls on the sand. Similarly, the nature of cotton balls and wool yarn changes when falling on the target. The qualitative aspects of the laws may be philosophical, but understanding is scientific when quantitatively studied, taking all possible factors into account. Thus, there is immense scope for research in this regard, and numerous experiments are possible. Likewise, values of other Qi's can be determined by comparative methods in specific experiments.

7 Applications of the Generalized Form in the Third Example of the Law

In view of the generalized form of the third law, i.e., eq. (18), the law of conservation of momentum or eq. (6) becomes:

$$\text{Change in motion of body B when body A impinges on it} = -Q \times \text{Change in motion of body A}$$

$$MV_{\text{final}} - MV_{\text{initial}} = -Q(mU_{\text{final}} - mU_{\text{initial}})$$
$$QmU_{\text{initial}} + MV_{\text{initial}} = QmU_{\text{final}} + MV_{\text{final}} \tag{23}$$

The coefficient Q accounts for various factors as described in eqs. (18-20), and the magnitudes of eq. (23) vary from eq. (6). Thus, experiments are necessary for a realistic understanding and measurement of Q. Also,

$$V_{gun} = -Q\frac{m_{bullet}v_{bullet}}{M_{gun}} \tag{24}$$

Further equations for ideal rocket equations become:

$$\text{Forward momentum of the rocket } (M\,dV) = -Q \times \text{ Backward momentum of the exhaust } (dMV_e) \tag{25}$$

The backward momentum of the exhaust (fire, gases, smoke, sparks, etc.) of the rocket (dMVe) should be more than the forward momentum of the rocket (Mdv). Only then will the rocket move upward; if both are equal, then forward movement is not possible; it is like a tug-of-war. or

$$V = QV_e ln(\frac{M_0}{M}) \tag{26}$$

This eq. (26) may be tested for fireworks under various conditions.

$$V = Qg_0 I_{sp} ln(\frac{M_0}{M}) \tag{27}$$

Now we have various types of flying vehicles, e.g., various types of modern rockets, jet engines, electric planes, solar-powered planes, autogyros, trikes, gliders, modern space crafts like Ingenuity, VSS Unity, etc. Ideally, the rocket equation can be tested for simple fireworks. Now, the motion of the rocket is controlled by a computer algorithm. The Falcon Heavy of Space X (re-useable rocket) is capable of landing back at designated points in the sea after flight [22]. Electric planes and solar-powered planes (the recent discoveries) do not emit exhaust (like rockets and fireworks), which is regarded as action. The gliders rise to an altitude of 76,000 feet without an engine or blades (propeller). Thus, gliders do not produce an exhaust or action force like rockets, but even then, move forward. Newton's third law requires an action force (as an exhaust in the case of rockets or fireworks), thus reaction is produced. Now, various aspects relating to this and eq. (27) require separate discussion [23].

7.1 The Equations in a One-Dimensional Elastic Collision When Generalized for Momentum

In view of the generalized form of Newton's third law, i.e., eq. (18), eqs. (14), (15) become:

$$U_{final} = \frac{[Q^2 m - M]U_{initial} + 2QMV_{initial}}{Q^2 m + M} \tag{28}$$

$$V_{final} = \frac{V_{initial}[M - Q^2 m] + 2mU_{initial}}{[M + Q^2 m]} \tag{29}$$

The eqs. (14)-(15) indicate that the final velocity of the projectile and the final velocity of the target do not depend upon the shape of the body, characteristics of the body, nature of the surface, etc.; the eqs. (28), (29) indicate that these factors are very significant and taken into account by the coefficient Q. The coefficient Q can be determined experimentally. Like eqs. (16), (17), the initial velocities of the projectile and target can be calculated from eqs. (28), (29):

$$U_{initial} = \frac{U_{final}[Q^2 m + M] - 2QU_{initial}}{[Q^2 m - M]} \tag{30}$$

$$V_{initial} = \frac{V_{final}[M + Q^2 m] - 2mU_{initial}}{[M - Q^2 m]} \tag{31}$$

7.2 Compelling Theoretical Requirements to Experimentally Confirm the Equations

Now, in this case, the denominators of eq. (30) and (31) do not become zero when the mass of the projectile is equal to the mass of the target (m=M); like eq. (16) and eq. (17). It is due to the presence of the additional factor Q. The value of Q is such that the denominators $[Q^2 m - M]$ and $[M - Q^2 m]$ are non-zero, as Q is an additional variable in eqs. (30),(31). There are no such factors in eqs. (16)-(17), based on the original form of the third law of motion. So, this is another advantage of the generalized form of Newton's third law of motion. Thus, inconsistency leading to the Special Theory of Relativity can be avoided, theoretically. However, the value of Q can be experimentally determined; also, equations based on the original form of the third law of motion are yet to be experimentally confirmed.

8 Conclusions

Even at the qualitative level, factors like shapes (a spherical, semi-spherical, long pipe, flat, irregular, cone, distorted shape, or any feasible typical shape, etc.) and characteristics of bodies (for rubber, superball, sky ball, etc.) play significant roles in the motion of bodies. The bodies may be of various types. Newton's third law, i.e., eq. (1), is universally and absolutely applicable and does not take these factors into account. However, these can be taken into account in the generalized form: Reaction = - Action ($Q_{shape} \times Q_{composition} \times Q_{target} \times Q_{other}$). The eqs. (18-20) can be experimentally confirmed. It can also be used in other cases where the law is applied, e.g., in elastic collisions, the Tsiolkovsky equation, etc.

This study does not affect the already established status of Newton's third law of motion but theoretically extends its conceptual basis and applications with logical discussion. There is sufficient theoretical and conceptual evidence that the various equations based on the third law must be tested experimentally in a quantitative way.

9 Declarations

There is no conflict of interest of any type with the manuscript.

10 Acknowledgments

The author is highly thankful to Professor Gordon Ramsey, President AAPT, Dr. Stephan Crothers, and various anonymous reviewers, including Dr. V P S Awana, Professor B C Chauhan, Professor V S Rangra, Dr. Amarjeet, Dr. Dinesh Kumar, and Dietician Anjana Sharma, for constant encouragement at various stages of this work.

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