Why is a 60° prism preferred for observing dispersion

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Abstract

Prisms are commonly used in classrooms to study refraction and dispersion, with 60° prisms being the most prevalent in school and college laboratories. In this article, we explore the reasons behind their widespread use. Using Cauchy's law, we determine the refractive indices for violet and red light for a selected prism. Applying Snell's law and the principle of total internal reflection, we demonstrate that a 60° prism provides optimal dispersion, maximizing the separation between red and violet rays over a broad range of incidence angles.

1 Introduction

In 1666, Sir Isaac Newton [1] investigated the effect of passing light through triangular glass prisms with different refracting angles. His experiments led to the conclusion that white light consists of a spectrum of colors. Over time, extensive studies on dispersion led to new techniques for determining refractive indices [2, 3, 4, 5, 6]. Despite these advancements, equilateral prisms remain the standard in school and college laboratories, with textbooks consistently depicting 60° prisms in sections on refraction [7, 8]. However, the reasoning behind this preference is unclear. Do these prisms offer superior dispersion? In this article, we explore this question, beginning with a review of refraction through a prism in the next section.

2 Refraction through a prism

In Figure 1, we can see that A is the angle of the prism, i_1 is the angle of incidence, i_2 is the angle of emergence, r_1 is the angle of refraction at the first side and r_2 is the angle of refraction for the second side. For refraction of light through a prism we have $r_1 + r_2 = A$ and deviation[7]

$$d = i_1 + i_2 - A \tag{1}$$

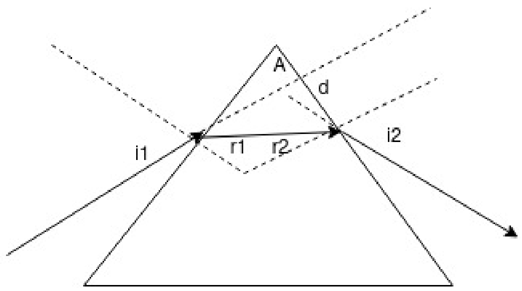

Figure 1: Refraction through a prism

From Snell's law we have $\sin i_1 = \mu \sin r_1$ and $\sin i_2 = \mu \sin r_2$, where μ is the refractive index of the prism. Then substituting for i_2 and r_2

$$d = i_1 + \sin^{-1}\left[\mu \sin(A - r_1)\right] - A$$

Finally substituting the value of r_1, the angle of deviation

$$d = i_1 + \sin^{-1}\left[\mu \sin(A - \sin^{-1}\left(\frac{\sin i_1}{\mu}\right)\right] - A \quad (2)$$

The Equation(2) shows that deviation depends only on refractive index, angle of incidence and angle of the prism. Next we will obtain refractive index for some colors.

3 Refractive index for red and violet rays

For each color we have different refractive index given by Cauchy's law[4]

$$\mu = A + \frac{B}{\lambda^2}$$

where A and B are Cauchy's constants. By using minimum deviation method, we can find A and B and then μ for different λ. Using a prism available in the laboratory, we got the refractive index for red and violet light as

$$\mu_r = 1.40 \quad (3)$$

$$\mu_v = 1.58 \quad (4)$$

respectively. These values will be different for different prisms. We are interested in finding the deviation of red and violet rays which are related with μ. Hence we found μ only for red and violet. Next we will find whether for any A and angle of incidence, dispersion is possible.

4 Total internal reflection and absence of dispersion for some angle of incidences

When the light rays travel from an optically denser medium to a rarer medium, for a particular angle of incidence called critical angle C given by the relation $\sin C = \frac{1}{\mu}$ is reached, the ray will be totally internally reflected. We will check whether this will happen for red and violet rays for different angled prisms. Using the μ given by Eqn(3) and (4) the critical angle for red and violet are

$$C_{red} = 45.58^\circ$$

$$C_{violet} = 39.26^\circ$$

Using the expression $r_1 = \sin^{-1}\left(\frac{\sin i_1}{\mu}\right)$ we can get the refraction angle for the first surface and using $r_1 + r_2 = A$, we can find the refracting angle for the second surface r_2. Let us check whether r_2 matches with the critical angle values for different angle of incidences. We will find this for $i = 30^\circ, 35^\circ, 40^\circ$and 60° for different A's.

4.1 For $i = 40^\circ$

From the Table 1 we can observe that when

Table 1:

A in degree	$\mu = 1.4$ r_2 in degree	$\mu = 1.58$ r_2 in degree
10	-17.33	-18.14
20	-7.33	-8.14
30	2.67	1.86
40	12.67	11.86
50	22.67	21.86
60	32.67	31.86
70	42.67	**41.86**
80	**52.67**	**51.86**

A is 70°, the violet ray gets totally internally reflected and hence there will be no dispersion. The negative sign in the refracted angle for small prisms is due the refraction to the other side of the normal compared to the side of the incident ray.

4.2 For $i = 60^\circ$

The value of r_2 for different angles of the prism are given in Table 2. Here violet light gets totally internally reflected for 80° prism. We see that for larger angle of incidence the total internal reflection happens for larger angle prisms. So let us go for low angle of incidence.

Table 2:

A in degree	$\mu = 1.4$ r_2	$\mu = 1.58$ r_2
10	-28.21	-23.24
20	-18.21	-13.24
30	-8.21	-3.24
40	1.79	6.76
50	11.79	16.76
60	21.79	26.76
70	31.79	36.76
80	41.79	**41.76**

Table 3:

A in degree	$\mu = 1.4$ r_2 in degree	$\mu = 1.58$ r_2 in degree
10	-14.21	-11.1
20	-4.21	-1.1
30	5.71	8.9
40	15.71	18.9
50	25.71	28.9
60	35.71	38.9
70	**45.71**	**48.9**
80	55.71	58.9

4.3 For $i = 35^\circ$

We can see that the total internal reflection happens for both violet and red for 70° prism. Usually we take angle of incidence between 30° and 60°. So we find what happens at = 30°, which is given in Table 4.

4.4 For $i = 30^\circ$

When we take $i = 30^\circ$ we see that for $A = 60^\circ$ there is no dispersion.

Table 4:

A in degree	$\mu = 1.4$ $r_2$in degree	$\mu = 1.58$ r_2 in degree
10	-11.1	-8.66
20	-1.1	1.34
30	8.9	11.34
40	18.9	21.34
50	28.9	31.34
60	38.9	**41.34**
70	**48.9**	51.34
80	**58.9**	**61.34**

5 Observations from the above studies

Above studies show that for getting dispersion without total internal reflection upto 60° prism, we have to choose the angle of incidence between $i = 30^\circ$ and $i = 60^\circ$. Next we will find for which angled prism we get maximum dispersion with the above choice of angle of incidences.

6 Separation between violet and red rays

From Eqn(2) we can find deviation for violet, d_v and deviation for red d_r and to get a maximum separation, $d_v - d_r$ must be large. So we found $d_v - d_r$ for different i and A. We found that $d_v - d_r$ is maximum for a 60° prism, which ensures maximum dispersion. This is shown in Table 5

Table 5:

A in degrees	$d_v - d_r$ in degrees
10	2.1
20	3.95
30	5.76
40	7.93
50	11.17
60	19.13
70	Absent
80	Absent

7 Conclusions

In typical laboratory experiments, the angle of incidence is chosen between 30° and 60°. Our analysis demonstrates that, within this range, a 60° prism provides the best dispersion, ensuring well-separated red and violet rays. This explains why equilateral prisms are widely used in educational settings for studying refraction and dispersion. Their optimal performance in practical conditions makes them the preferred choice for instructional experiments.

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