Introducing Raman's study of drums in laboratories

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Submitted on 19-10-2021

Abstract

In many college laboratories across India, students conduct or replicate experiments performed by scientists worldwide. However, the groundbreaking experimental work of Sir C. V. Raman, one of India's most brilliant experimentalists, is often overlooked. To address this gap, we propose a simple and cost-effective method to introduce Raman's classic experiment on drums into college laboratories.

1 Introduction

Sir C. V. Raman's study about drums was first published in 1920, while he was at Kolkata [1]. In 1922 he gave a detailed account on the Indian contribution of making musical drums [2]. After 15 years of detailed studies, while in Bangalore, he published a breakthrough paper on the harmonics generated by Mridangam [3], which is actually the beginning of a systematic study of musical drums. Out of many musical drums in India, Mridangam is the most ancient instrument and it is played in concerts and art forms. Many theoretical and experimental studies about the vibrations of the drum head of Mridangam were published later [4–7]. Our paper gives a simple method for the study of harmonics in musical drums in an undergraduate laboratory very easily and cost-effectively. Before going to our method a brief idea about the contributions of Raman in the field of drums is given in the next section.

2 Raman's contribution

Raman, while working at Kolkata in Indian Financial Service, during his spare time, studied the acoustics of Indian drums [8].

The most important drum that attracted Raman with its highly harmonious tones was Mridangam. One of the objectives of Raman on studying was to find the differences in the construction of the Indian musical instruments from other regions of the world and to find the most important component that contributes to the tonal qualities of Indian instruments. Raman found that the

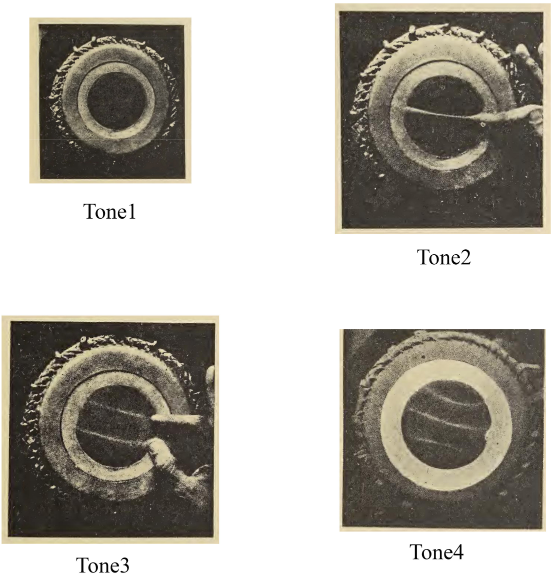

Figure 1: The production of tones on Mridangam head as in Reference [9]

black paste on the drum head is the cause of shifting the non-harmonic overtones in a circular membrane to harmonic tones in Indian instruments like Mridangam. To excite different harmonic tones, Raman sprinkled lycopodium power on the drum head and the drum was played with experienced artists [10]. By hearing and seeing the tones produced, Raman identified the harmonics as there were no modern techniques for study at that time. The images of Mridangam drum head excited by Raman are given in Figure 1. The frequency ratios obtained by Raman [3] are given in Table 1

Mode	Ratio
0,1	1.000
1,1	2.00
2,1	3.00
0,2	3.25
3,1	4.00
1,2	4.321
4,1	5.00
2,2	5.34
0,3	5.167

Table 1

This is a historical discovery that was published in Nature [1] a prestigious journal of physics. Before going to the details of our study we will have an understanding of harmonics and their relevance in music. What is meant by a mode will be discussed in another section.

3 Harmonics

A sound produced by any instrument consists of a collection of frequencies. The lowest frequency among them is termed as the fundamental frequency and higher frequencies are termed as overtones. These overtone frequencies and the fundamental are called together as harmonics when they have an integer multiple number relation. Consider the set of frequencies in Table 2.

Frequency(Hz)	Ratio
150	1
210	1.4
260	1.73
300	2
330	2.2
450	3
600	4

Table 2

Here the lowest frequency is 150Hz and hence it is the fundamental frequency and all other frequencies are called overtones of fundamental. But only 300Hz, 450Hz, 600Hz are harmonics along with fundamental since they have integer ratios. Hence the 150 Hz is called first harmonic, 210Hz and 260 Hz are called first and second overtones but not harmonics as they do not form integer ratio with 150 Hz. The 300Hz is called third overtone and second harmonic and so on. The central loaded region of the Mridangam is called the **Karane**(Figure 2).

Figure 2: The black loading on Mridangam head

4 Modes of vibration of a simple circular membrane

In India many drums like Dhol, Timila etc use one or more layers of animal skin stretched over a wooden shell and are played with either hand or stick. The vibrations of a single layer of animal skin attached to the drum head are studied as a circular membrane with a fixed boundary. A mode is a pattern of vibration in which the whole membrane vibrates except certain points that remain at rest called nodes. The two-dimensional wave equation represents the vibration of a circular membrane [11] is.

$$\nabla^2\psi(r,\theta) = \frac{1}{c^2}\frac{\partial^2\psi(r,\theta)}{\partial t^2}$$

Here $\psi(r,\theta)$ is the transverse displacement, r is radial component of displacement and θ is the angular component of displacement and $c = \sqrt{\frac{T}{d}}$ where T is the tension produced on the membrane and d is the mass density. The r-dependent solution for the membrane is [12]

$$\psi(r) = AJ_n(kr) + BY_n(kr)$$

where $J_n(kr)$ is the Bessel function of first kind with order n and $Y_n(kr)$ is the Bessel function of second kind with order n. At the center of the membrane $r = 0$. The Bessel function of second kind has no finite solution at the centre. Hence we consider only first part of the solution as

$$y(r) = AJ_n(kr)$$

At boundary $r = a$, since it is fixed we get

$$J_n(ka) = 0$$

There are numerous solutions for this equation given by x_{nm}. These solutions are termed as zeros of the Bessel function or roots of the Bessel function. The number of the roots is represented by m and the order of the Bessel function is represented by n. Thus we get

$$ka = x_{nm}$$

Physically on a drum head, the non-vibrating points originate along the diameter or along the circumference of a circle called nodes. The straight line and the circle created by zero vibration points are then called nodal diameter and nodal circle respectively. The number of nodal diameters

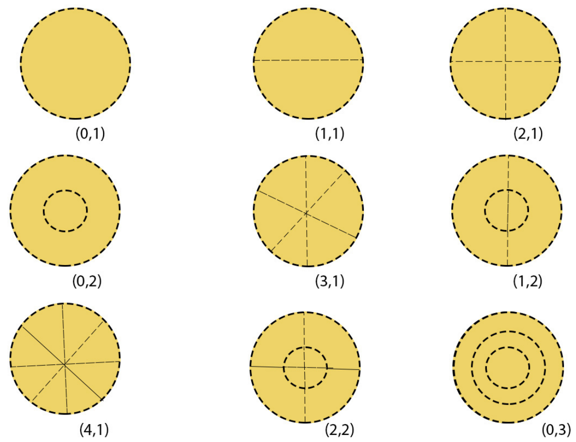

Figure 3: The modes of vibration of circular membrane

formed on a drum head is represented by the order of the Bessel function n and the number of nodal circles formed on the drum head are indicated by the number of roots of the Bessel function m. The first few modes of vibration in circular membrane are shown in Figures 3 and 4. The frequency ratios

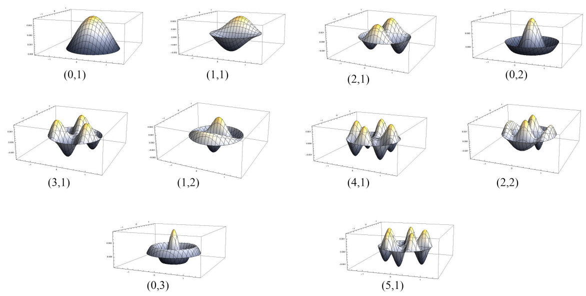

Figure 4: The modes of vibration of circular membrane

of first few modes of vibration of a circular membrane calculated using the zeros of the Bessel function [13] are given in the Table 3

Mode	Ratio
0,1	1.00000
1,1	1.59335
2,1	2.13556
0,2	2.29545
3,1	2.65311
1,2	2.91733
4,1	3.15548
2,2	3.50016
0,3	3.59851
5,1	3.64749

Table 3

The lowest mode is the fundamental mode which has only one nodal circle and no nodal diameter. The axis-symmetric vibration of modes do not generate nodal diameters and hence all modes with $n = 0$ have only nodal circles. The first nodal diameter is formed by the vibration of the second mode that has one nodal circle also. This mode generates the first overtone frequency which is 1.5933 times the fundamental. Out of nine overtone generating modes given in the table, second, third, fifth, seventh and tenth modes have one nodal circle and 1, 2, 3, 4 and 5 nodal lines respectively. Two nodal circles are formed in the fourth, sixth and eighth modes with 0,1, and 2 nodal lines respectively. The changes in the vibration in angular direction generates these nodal diameters. The maximum number of nodal circles are formed by the ninth mode with no nodal diameter. From Table 3, it can be seen that all the overtones have a non-integer ratio with fundamental and due to this none of them form harmonics. Thus, the sound produced by an ordinary circular membrane drum head is not musical. Now let us study the music produced by Mridangam in the next section.

5 Music produced by Mridangam

Mridangam is a drum built with wood and air is enclosed inside the instrument after construction [14]. A Mridangam is shown in Figure 5. A black region with high density is made on the drum head constructed with goat skins by rubbing fine paste made of rice, and stone that is known 'Purana-keedam'. The cow or buffalo hide is used to make an annular region on the drum head and ropes are used to stretch the drum head tightly on the rim. The circular gap between the loaded region and annular region is filled with materials like tiny plastic balls, dried stems of wheat or paddy. Harmonic tones are produced by five strokes on the right loaded head in Mridangam. The strokes include Dheem, Arachappu, Naam, Chappu and Dhin [15]. The stroke Dheem is produced by striking the loaded region on the drum head with fingers in the right hand and removing immediately. The Arachappu is produced by the little finger, strikes along the diameter in the loaded region away from the centre and other fingers are used as support. Naam stroke is made by striking at the edge of the annular membrane with for finger and keeping the dark region at rest by placing the ring finger at its circumference. The Chappu is elicited by striking with the little finger at the circumference of the loaded region towards the centre. The final stroke Dhin is produced by striking fore finger at slightly inward to the circumference of the loaded region and the ring finger is placed on the circumference at 60-degree distance.

Figure 5: The Mridangam

6 Our method

The audio samples of Mridangam strokes in MP3 format with a duration of few seconds are collected. The samples are placed in a folder on a mobile phone. For analysis, the **Visual analyzer** software freely available on the internet is used. The installation is done by double-clicking on the executable file and following the instructions. Open the software using the icon on the desktop. A screenshot of the software window is shown in Figure 6. Connect headset or mike on the

Figure 6: The screenshot of Visual Analyzer software

laptop, place the mobile phone near the talking point of the headset. Click on the **ON** button in the menu bar before the settings menu and play the sound from the mobile phone. The capture spectrum button on the panel in the left bottom end in the software window. A new window showing the frequency spectrum will appear. Left-click and drag to zoom the spectrum at the beginning of the axis. Place the mouse at each peak in the frequency spectrum and corresponding x-axis and y-axis values are seen at the bottom. The spectrum is saved as a PDF file using

$$File \longrightarrow Printspectrum$$

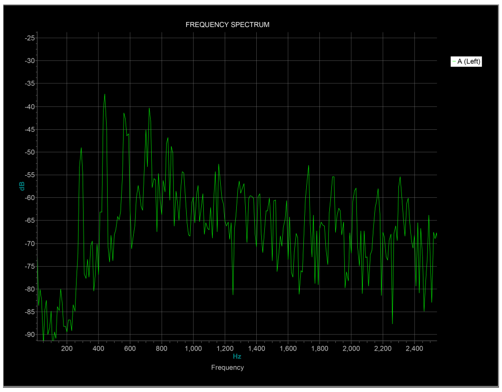

Figure 7: The frequency spectrum of Naam stroke

7 Results

The Dheem stroke excites the lowest mode or fundamental mode. For all other strokes, the fundamental is slightly higher than one. This indicates that the other strokes do not vibrate in fundamental mode since the fingers placed on the drum head suppress it. In such strokes, the higher harmonics that are in the integer relation are identified by the brain and generates a pitch sense around the frequency of the second harmonic present in the stroke. To tune the drum head, artists use Arachappu or Naam strokes. Hence we plot Naam stroke as an example that is given in Figure 7. Here we used a Mridangam tuned to pitch D3 and its standard frequency is 146.83Hz. The frequency of the peaks present in the spectrum is given in Table 4.

Number of peak	Frequency (Hz)	Ratio
1	160.059	1.10
2	290.61	2.00
3	439.169	3.02
4	560.717	3.85
5	720.53	4.95

Table 4

8 Discussion

Here, the third peak is the most prominent, indicating that the third harmonic tone is heard when the Naam stroke is excited. Similarly, the most prominent peak in the spectrum is used to determine the order of the harmonic tone produced. In other words, the modes vibrating with maximum energy are identified from the dominant peak.

C. V. Raman extensively studied the acoustics of the Indian drum Mridangam and published his findings after years of meticulous evaluation. Today, the same tonal characteristics can be analyzed with significantly less effort and time using modern software techniques. The study of musical drums and their harmonics can serve as an engaging experiment for undergraduate students, helping them understand and appreciate C. V. Raman's contributions to the field of acoustics.

9 Conclusions

We believe that the best way to honor Raman, India's greatest scientist, is by studying and replicating his experiments. His research is so insightful that dedicating a section of college laboratories to exploring his contributions would be highly beneficial. In this paper, we demonstrate that Raman's study of harmonics can be easily conducted using a computer, a resource readily available in most college laboratories.

10 Acknowledgments

The authors wish to express their gratitude to Mr. Arjun, a skilled Mridangam player, for his invaluable assistance in identifying, distinguishing, and recording the various tones produced by the instrument.

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