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## Editorial: Advancing Foundations and Frameworks in Physics Education

The current issue of the *Journal of Physics Education* brings together a diverse and thought-provoking collection of papers that span the foundational pillars of statistical mechanics, thermodynamics, and the evolving role of emerging technologies in physics pedagogy. By addressing both the deep conceptual frameworks of physics and the tools we use to teach and understand them, these contributions offer valuable insights for educators, researchers, and students alike.

### Navigating the Foundations of Statistical and Thermal Physics

A significant portion of this issue is dedicated to demystifying the complex, often abstract concepts that form the bedrock of statistical mechanics and thermodynamics.

- **A Fresh Pedagogical Approach to Khinchin:** Complementing these studies, Prasanth P., P. Aneesh Kumar, and K. M. Udayanandan provide an accessible entry point into a rigorous mathematical framework with "**Khinchin's Statistical Mechanics: A Concise Introduction**" (pp. 1–6). The authors successfully distill complex mathematical foundations into a streamlined guide, making the information-theoretic approach to statistical mechanics highly approachable for classroom instruction.
- **Establishing Conceptual Baselines:** In "**PER Study of Fundamental Concepts in Statistical Physics-I: Concept inventory on Microstate, Macrostate and Steady State**" (pp. 1–23), authors Vandana Sharda, Jyoti Bhardwaj, O.S.K.S. Sastri, and Arbind K Jha present a crucial Physics Education Research (PER) study. By developing a dedicated concept inventory, the authors provide educators with a rigorous tool to assess and address student understanding of foundational ideas like microstates, macrostates, and steady states, paving the way for more effective classroom interventions.
- **Looking to the Future of Educational Technology:** Beyond foundational physics, this issue also looks forward to the rapidly changing landscape of educational tools. In "**Outlook for ET Intelligence Lookout**" (pp. 1–14), A. Amitvikram Sharma, B. Pranav Sharma, C. Jayavrinda Vrindavanam, and P.C. Deshmukh offer a forward-looking analysis of educational technology (ET). Their work examines how intelligent frameworks can be leveraged to monitor, assess, and enhance learning outcomes, providing a timely perspective as digital platforms become increasingly integrated into physics curricula worldwide.
- **The Mathematical Rigor of Entropy:** Shifting focus to thermodynamic consistency, P. Reshma, P. Prasanth, S. Bhagyasree, and K. M. Udayanandan explore the core properties of thermal systems in "**Necessary Conditions for the Extensivity of Entropy**" (pp. 1–3). This concise yet impactful paper clarifies the essential parameters required to maintain entropy as an extensive property, a vital concept for advanced undergraduate and graduate physics curricula.

Together, these articles remind us that advancing physics education requires a dual commitment: continually refining how we teach the timeless, foundational laws of nature, while eagerly embracing and evaluating the new technological frontiers that shape the classrooms of tomorrow. We hope this issue inspires fresh discussions, refined teaching strategies, and further research in your own classrooms and institutions.

### Editor-in-Chief

*Journal of Physics Education*

# Khinchin's Statistical Mechanics: A Concise Introduction

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## Abstract

Khinchin's formulation, which does not depend on the Ergodic hypothesis, plays a crucial role in the foundations of statistical mechanics. In this article, we briefly discuss about this formulation and demonstrate how the thermodynamics of various systems can be derived using this formulation.

## 1 Introduction

Statistical mechanics (SM) is a fundamental branch of physics that explains temperature-dependent properties of systems. It has two primary formulations: Boltzmann's theoretical approach and Gibbs' more practical treatment. Both rely on the Ergodic hypothesis, which asserts that phase and time averages are equal for any dynamical system—an assumption that remains unproven. In 1930, G. D. Birkhoff [1] intro-

duced Ergodic theory to establish this equality based on system dynamics, but proving it remained challenging. Khinchin [2] later addressed the problem differently, arguing that Ergodic theory was too general and lacked specific applicability to typical systems in statistical mechanics. Khinchin argued that solving the Ergodic problem must be linked to the system's characteristics. He made a new approach which relied mainly on the assumption: Phase functions are sum functions, expressed as

$$f = \sum_i^n f_i$$

where  $f_i$  represents the phase function of each particle. An example is the Hamiltonian  $H$  of a non relativistic free particle system:

$$H = \sum_i^N \frac{p_i^2}{2m}$$

where  $p_i$ 's are the momenta of particles and  $m$ , the mass.

Phase functions are named so because they are functions of the phase space variables of a system. In classical mechanics, the phase space is defined by the set of all possible states of a system, where each state is specified by the generalized coordinates  $q_i$  and generalized momenta  $p_i$ . Since macroscopic properties are sums of individual contributions, statistical averages become simple sums over all particles. Instead of analyzing the full many-body phase space, we only need to consider the behavior of individual particles. Physical quantities like energy, momentum, and entropy should be extensive, meaning they scale with the number of particles. If  $f$  is a sum function, it naturally satisfies extensivity:

$$f(N) \propto N$$

In traditional statistical mechanics (e.g., Boltzmann's approach), the **Ergodic hypothesis** is used to justify averaging over phase space. Khinchin's assumption of sum functions provides an alternative: since macroscopic properties are sums of many independent contributions, statistical fluctuations become negligible for large  $N$ , leading to well-defined thermodynamic behavior. One of the limitation of this assumption is that it does not hold for systems with strong interactions, such as those with long-range forces (e.g., gravitational systems, strongly correlated quantum systems).

## 2 Khinchin Formalism

In this section we will explain the formalism in a brief way.

Khinchin's method provides a statistical approach to determining thermodynamic properties. This formalism is based on three key mathematical constructs:

- The **phase space volume**  $\Gamma(E)$ , which defines the accessible micro states for a system at energy  $E$ .
- The **structure function**  $\Omega(E)$ , which represents the density of micro states given by

$$\Omega(E) = \frac{\partial \Gamma(E)}{\partial E}. \quad (1)$$

- The Laplace transform of  $\Omega(E)$ , known as the **generating function**  $\phi(\alpha)$ , is given by:

$$\phi(\alpha) = \int_0^{\infty} \Omega(E) e^{-\alpha E} dE. \quad (2)$$

Here,  $\alpha$  is related to the inverse temperature by:

$$\alpha = \frac{1}{k_B T}. \quad (3)$$

### 2.1 Thermodynamics

#### 2.1.1 Energy

The mean energy  $\langle E \rangle$  is given by:

$$\langle E \rangle = \frac{\int_0^{\infty} E \Omega(E) e^{-\alpha E} dE}{\int_0^{\infty} \Omega(E) e^{-\alpha E} dE}. \quad (4)$$

Using the definition of  $\phi(\alpha)$ , this simplifies to:

$$\langle E \rangle = \frac{\int_0^\infty E e^{-\alpha E} \Omega(E) dE}{\phi(\alpha)}. \quad (5)$$

Now, we differentiate  $\phi(\alpha)$  with respect to  $\alpha$ :

$$\frac{d}{d\alpha} \phi(\alpha) = \int_0^\infty \frac{d}{d\alpha} (e^{-\alpha E}) \Omega(E) dE. \quad (6)$$

Since

$$\frac{d}{d\alpha} e^{-\alpha E} = -E e^{-\alpha E}, \quad (7)$$

we obtain

$$\frac{d}{d\alpha} \phi(\alpha) = - \int_0^\infty E e^{-\alpha E} \Omega(E) dE. \quad (8)$$

Dividing both sides by  $\phi(\alpha)$ , we get:

$$\frac{1}{\phi(\alpha)} \frac{d}{d\alpha} \phi(\alpha) = - \frac{\int_0^\infty E e^{-\alpha E} \Omega(E) dE}{\phi(\alpha)}. \quad (9)$$

Recognizing that the right-hand side is exactly the expectation value  $\langle E \rangle$ , we obtain:

$$\langle E \rangle = - \frac{d}{d\alpha} \ln \phi(\alpha). \quad (10)$$

From  $\langle E \rangle$ , temperature T can be obtained.

## 2.2 Entropy

Khinchin proposed that entropy should be expressed as:

$$S = k_B \ln \phi(\alpha) + k_B \alpha \langle E \rangle. \quad (11)$$

Here:

- $k_B \ln \phi(\alpha)$  is similar to the standard entropy definition in statistical mechanics.
- The additional term  $k_B \alpha \langle E \rangle$  ensures consistency with thermodynamics.

To obtain a more compact expression, we define a **modified generating function**:

$$\phi_a(\alpha) = e^{\alpha \langle E \rangle} \phi(\alpha). \quad (12)$$

Taking the natural logarithm:

$$\ln \phi_a(\alpha) = \ln \phi(\alpha) + \alpha \langle E \rangle. \quad (13)$$

Substituting into the entropy expression:

$$S = k_B \ln \phi_a(\alpha). \quad (14)$$

This results in a **single logarithmic term**, making the entropy formula more compact. This ensures that entropy is now expressed entirely in terms of  $\phi_a(\alpha)$ , simplifying its mathematical form.

## 2.3 Pressure

The mean energy is given by:

$$\langle E \rangle = - \frac{d}{d\alpha} \ln \phi(\alpha, V). \quad (15)$$

From thermodynamics, pressure is defined as:

$$P = - \left( \frac{\partial \langle E \rangle}{\partial V} \right)_\alpha. \quad (16)$$

Substituting the expression for  $\langle E \rangle$ :

$$P = - \frac{\partial}{\partial V} \left( - \frac{d}{d\alpha} \ln \phi(\alpha, V) \right). \quad (17)$$

Simplifying:

$$P = \frac{\partial}{\partial V} \left( \frac{d}{d\alpha} \ln \phi(\alpha, V) \right). \quad (18)$$

which can be simplified to

$$P = \frac{1}{\alpha} \frac{\partial}{\partial V} \ln \phi(\alpha, V). \quad (19)$$

### 3 Applying Khinchin formalism

Let us use the above thermodynamic equations and find energy, entropy and pressure for any dimensional system.

#### 3.1 Generalized Micro states

Consider a system where the number of accessible micro states is given by [3]:

$$\Gamma(E) = CV^m E^l. \quad (20)$$

Taking the derivative to obtain the structure function:

$$\Omega(E) = \frac{d\Gamma}{dE} = CmlV^m E^{l-1}. \quad (21)$$

The generating function is defined as the Laplace transform of  $\Omega(E)$ :

$$\phi(\alpha, V) = \int_0^\infty \Omega(E) e^{-\alpha E} dE. \quad (22)$$

Substituting  $\Omega(E)$ :

$$\phi(\alpha, V) = CmlV^m \int_0^\infty E^{l-1} e^{-\alpha E} dE. \quad (23)$$

Using the standard integral result:

$$\int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma(n)}{a^n}, \quad \text{for } a > 0, \quad (24)$$

we get:

$$\phi(\alpha, V) = CmlV^m \frac{\Gamma(l)}{\alpha^l}. \quad (25)$$

#### Mean Energy

From Khinchin's formalism, the mean energy is:

$$\langle E \rangle = -\frac{d}{d\alpha} \ln \phi(\alpha, V). \quad (26)$$

Substituting  $\phi(\alpha, V)$ :

$$\ln \phi(\alpha, V) = \ln C + \ln(mlV^m) + \ln \Gamma(l) - l \ln \alpha. \quad (27)$$

Taking the derivative:

$$\langle E \rangle = -\frac{d}{d\alpha} (-l \ln \alpha) = \frac{l}{\alpha}. \quad (28)$$

Using  $\alpha = \frac{1}{k_B T}$ , we obtain:

$$\langle E \rangle = lk_B T. \quad (29)$$

#### Entropy

The entropy is given by:

$$S = k_B [\ln \phi(\alpha, V) + \alpha \langle E \rangle]. \quad (30)$$

Substituting values:

$$S = k_B [\ln C + \ln(mlV^m) + \ln \Gamma(l) - l \ln \alpha + l]. \quad (31)$$

Replacing  $\alpha = 1/(k_B T)$ :

$$S = k_B [\ln C + \ln(mlV^m) + \ln \Gamma(l) + l \ln(k_B T) + l]. \quad (32)$$

**Pressure**

The pressure is obtained from:

$$P = \frac{\partial}{\partial V} \left( \frac{d}{d\alpha} \ln \phi(\alpha, V) \right). \quad (33)$$

simplifying

$$P = \frac{1}{\alpha} \frac{\partial}{\partial V} \ln \phi(\alpha, V). \quad (34)$$

From:

$$\ln \phi(\alpha, V) = m \ln V + (\text{constants}) \quad (35)$$

we get:

$$\frac{\partial}{\partial V} \ln \phi(\alpha, V) = \frac{m}{V}. \quad (36)$$

Thus:

$$P = k_B T \frac{m}{V}. \quad (37)$$

**3.2 Thermodynamics of an Ideal Gas**

Next we will use the above formulation and find out whether it agree with ideal gas thermodynamics. Let

$$\Gamma = V^N E^{3N/2}$$

We avoided C, the constant, because of its irrelevance in thermodynamics. From this, the **structure function** is:

$$\Omega(E, V) = \frac{d\Gamma}{dE} = NV^N E^{\frac{3N}{2}-1}. \quad (38)$$

The generating function is defined as the Laplace transform of the structure function:

$$\phi(\alpha, V) = \int_0^\infty \Omega(E, V) e^{-\alpha E} dE. \quad (39)$$

Substituting  $\Omega(E, V)$ :

$$\phi(\alpha, V) = NV^N \int_0^\infty E^{\frac{3N}{2}-1} e^{-\alpha E} dE. \quad (40)$$

Using the integral identity:

$$\int_0^\infty x^l e^{-\alpha x} dx = \frac{\Gamma(l+1)}{\alpha^{l+1}}, \quad (41)$$

we get:

$$\phi(\alpha, V) = NV^N \frac{\Gamma(\frac{3N}{2})}{\alpha^{\frac{3N}{2}}}. \quad (42)$$

**Mean Energy**

From Khinchin's formalism, the mean energy is:

$$E = -\frac{d}{d\alpha} \ln \phi(\alpha, V). \quad (43)$$

Taking the logarithm:

$$\ln \phi(\alpha, V) = N \ln V + \ln \Gamma \left( \frac{3N}{2} \right) - \frac{3N}{2} \ln \alpha. \quad (44)$$

Differentiating:

$$E = \frac{3N}{2} \frac{1}{\alpha}. \quad (45)$$

Using  $\alpha = \frac{1}{k_B T}$ , we obtain:

$$E = \frac{3}{2} N k_B T. \quad (46)$$

**Entropy**

The entropy in Khinchin’s formalism is given by:

$$S = k_B [\ln \phi(\alpha, V) + \alpha E]. \quad (47)$$

Substituting  $\phi(\alpha, V)$  and  $E$ :

$$S = k_B \left[ N \ln V + \ln \Gamma \left( \frac{3N}{2} \right) - \frac{3N}{2} \ln \alpha + \frac{3N}{2} \right]. \quad (48)$$

Using  $\alpha = \frac{1}{k_B T}$  and Stirling’s approximation

$$S \approx k_B \left[ N \ln V + \frac{3N}{2} \ln \frac{3N k_B T}{2} - \frac{3N}{2} \right]. \quad (49)$$

This is the **Sackur-Tetrode equation**.

**Pressure**

From Khinchin’s formalism:

$$P = \frac{1}{\alpha} \frac{\partial}{\partial V} \ln \phi(\alpha, V). \quad (50)$$

Since:

$$\ln \phi(\alpha, V) = N \ln V + (\text{other terms}), \quad (51)$$

differentiating w.r.t.  $V$ :

$$\frac{\partial}{\partial V} \ln \phi(\alpha, V) = \frac{N}{V}. \quad (52)$$

Thus,

$$P = \frac{1}{\alpha} \frac{N}{V}. \quad (53)$$

Using  $\alpha = \frac{1}{k_B T}$ , we obtain:

$$PV = N k_B T. \quad (54)$$

All the ideal gas thermodynamics defined earlier are obtained. [4]

**4 Conclusion**

Khinchin’s statistical mechanics, though challenging, offers a simplified approach under restrictive assumptions, such as treating the Hamiltonian as a sum function. This limits its applicability to systems with weak interactions and excludes phase transitions. However, it reinforces that statistical mechanics is valid only for systems with many degrees of freedom. Despite its constraints, we demonstrate that Khinchin’s method can effectively determine the thermodynamics of certain high-dimensional systems.

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# PER Study of Fundamental Concepts in Statistical Physics-I: Concept inventory on Microstate, Macrostate and Steady State

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## Abstract

**Background** Conceptual knowledge of microstate, macrostate and steady state is crucial to understanding Statistical Physics.

**Purpose** The objectives of present paper are: (i) To assess students' conceptual understanding/ difficulties on the ideas of microstate, macrostate and steady state using methodology of physics education research (PER), and (ii) To create a concept inventory, based on the above, to act as a validating tool

**Method** A total of 341 students at undergraduate and postgraduate level participated in

this study. Based on the students' responses to the interviews, open ended questions and semi-structured multiple choice questions, alternative conceptions were identified. These alternative conceptions were used as distractors in certain items of the concept inventory. Authors have created the concept inventory items keeping in mind the cognitive process knowledge dimensions of Revised Bloom Taxonomy. To minimize gains due to random guessing, a scoring key was devised via conceptual chronology of items. The concept inventory has been validated via face and content validation, item analysis and reliability tests performed on all the items.

**Result** A total of 18 items, in the form of multiple choice questions, were prepared and validated to form the original concept inventory. Then, it was divided into two equivalent concept inventories with 13 items in each. The five items which differ in two inventories were equivalent in terms of content relevance and difficulty level. Any of them could be used as pre-test or post-test.

**Conclusion** The PER based concept inventory can be used as a tool to measure students' difficulties on the chosen topics of microstate, macrostate and steady state. It can be utilised to evaluate relative effectiveness of different instructional strategies.

**Keywords** concept inventories; microstate; macrostate; steady state; thermodynamic; statistical physics. Physics Education Research(PER)

## 1 Introduction

Physics education researchers have considered various aspects of student reasoning that result in misconceptions and alternate conceptions of various physics concepts. Here, are a few important research based observations:

- “Students do not only bring ideas about how the physical world works into our classrooms. They also bring ideas about the nature of learning, the nature of science, and what it is they think they are expected to do in our class.”(pp.51). Redish opines that students knowledge and reasoning could be analysed in terms of their common naive

conceptions, primitive reasoning and the way it has been connected with real life situations.[1].

- According to Andrea Disessa, misconceptions are “fragmented collection of ideas, loosely connected and reinforcing”(pp.50). This can be viewed as intuitive physics, as it consists of large number of fragments called as phenomenological primitives, a simple abstraction of common experience [2].
- Moore in his book, *Science teaching reconsidered : A handbook*, categorized misconceptions as pre-conceived notions, non-scientific beliefs, conceptual misunderstanding, vernacular misconceptions and factual misconceptions[3].
- Berman figured out that, sometimes, elementary textbooks lead to misconceptions among students. This could be either due to inappropriate graphical presentation, communication gap or inappropriate use of words in a particular context. Fifteen misconceptions in elementary physics textbooks involving relativity, gravitation and cosmology have been identified[4]. Our own review of content on microstate, macrostate and steady state has confirmed this finding.
- Weiman[5] pointed out that, students' beliefs are crucial in teaching physics.
- Hamza[6] figured out that, Misconceptions emerged as alternatives due to questions that are not actively defended. Students'

reasoning evolved as a result of such specific situations.

- Docktor and Mestre described misconceptions traits as constructive in nature, oppose changes and interfere with scientific conceptions [7].

Then, the questions arose, as to how to identify misconceptions and the researchers have put a lot of effort and time into it. Here, we cite a few works:

- In a study by Morrison [8], the most experienced teacher was able to mention many examples of common preconceptions among students, where as, least experienced teacher provided examples which shows students' confusion among two terms. The study opined that, "In order to diagnose students' preconceptions, one needs to train students to participate in classroom discussion from very beginning when they start attending the lectures. Teacher must not only pose questions but should wait until students provide their answers. Given strategy requires sufficient amount of time and training of students as well as teachers".
- Researchers [9][10] [11] took interviews, paper and pencil test, used think aloud protocol and surveys to identify their misconceptions. .
- D.F. Styer has identified misconceptions based on the observations of students and colleagues write ups [12].

Finally, researchers have also put efforts into understanding how to handle these misconceptions:

- Gil [13] have shown that, constructivist model of science learning must be adopted to overcome misconceptions in science and other important problems of science learning.
- Stavy [14] identified that, in science, teaching by analogy is an effective tool in modifying misconceptions and learning new information.
- Thijis [15] specifically realised that, constructivist approach leads to the reduction in students misconceptions on the concept of force.
- Ryan and Aikenhead [16] have developed an instrument VOSTS in science education, that shows how science, technology, and society interact, school characterization of science and scientists, the social construction of scientific knowledge, and the epistemology of science.

## 2 Background

One of the foremost steps in identifying students naive conceptions is the creation of concept inventory. In science, the first diagnostic test as a multiple choice alternatives that has been applied to a large sample was by Halloun and Hestenes in 1985 [17], in their paper, 'Common sense concepts about motion'. In 1992, they formulated the Force concept inventory (FCI) [18], which became a benchmark in science education as it elaborated the whole process of development and validation of concept inventory. Till now, several concept inventories have already been formulated in science education. In Physics, concept

inventories have already been developed for the foundation courses such as mechanics [19]; [20] electricity and magnetism [21]; [22]; [23], quantum mechanics [24]; [25] and astronomy [26] [27]; [28]; [29].

This is for the first time that we have developed a multiple choice concept inventory on microstate, macrostate and steady state for the specialization course “*Statistical Physics*”. Before that, concept inventories on heat, temperature and thermodynamics have been developed for the students at 11<sup>th</sup> grade [30], at high school, college and University level [31]; [32], and at undergraduate engineering level [33]; [34]; [35]; [36]. They cover the basic concepts such as heat and temperature, thermodynamic laws, heat engines, heat pumps, Carnot cycles, thermal properties, entropy, reversibility, impossibility, Kelvin-Planck and Clausius statements.

Loverude and his students have done a lot of work in the field of Thermal and Statistical Physics. Diagnostic questions were formulated to probe students understanding of basic probability concepts. It was found that students find difficulty in distinguishing the concepts of microstate and macrostate of a system, and in relating mathematical relationships for multiplicity [37]. Students of upper-division courses in Thermal and Statistical Physics were examined to inquire their reasoning about entropy and the second law of thermodynamics. The finding suggested that students use a variety of conceptual resources rather than single simple resource of entropy which leads to contradiction in some cases. [38]. Smith et.al. have created a guided Inquiry worksheet tutorial to help

students in understanding the concepts of Boltzmann distribution and Canonical partition function. It was found that students were benefitted from the tutorial [39]. The guided inquiry worksheet has also been created for the tutorial on the concepts of Carnot efficiency and Carnot cycle as an upper level thermodynamical course. Results have shown that most of the students gained clarity on the concepts taught [40]. Simulations in Scilab were created to explain the concepts of the Maxwell Boltzmann speed and velocity distribution, and Joule’s free expansion of a gas [41]. Two inquiry based activities have been formulated for concepts: entropy, reversibility, confusion between enthalpy and internal energy, confusion between equilibrium and steady state, and confusion over factors impacting chemical equilibrium and rate [42]. Open source physics java programs were formulated on coin flip equilibrium and ideal gas expansion to demonstrate the concepts of probability, entropy and second law of thermodynamics [43]. The demon algorithm was created to help students understand the concepts of temperature and chemical potential [44]. We have developed a worksheet based simulation activity for teaching steady-state equilibrium. [45]

Methodology adopted in this paper is described in the next section. It involves the process of shortlisting the concepts as well as building the research questions for the paper. Further, formulation and validation of concept inventory is discussed in two separate subsections. Finally, we conclude and recommend some suggestions for further research.

### 3 Methodology

In the present paper, methodology given by Wendy K. Adams and Carl E. Wieman has been adopted[46].

#### 3.1 Shortlisting the Basic concepts of Thermodynamic and Statistical Physics

The experts teaching ‘Thermodynamics and Statistical Physics’ or ‘Statistical Physics’ at undergraduate or postgraduate level were requested to list the five most important concepts which they would like their students to have firm knowledge after they have completed the course. Moreover, the syllabus of chosen subject at undergraduate course content of various Universities have also been examined. Thence, the shortlisted basic concepts of Thermodynamics and Statistical Physics are as follows:

- Kinetic Theory of Gases
- Laws of Thermodynamics
- Probability Calculations and Statistical distributions
- Microstate and Macrostate of a system
- Entropy

#### 3.2 Identification of Alternative Conceptions/ Misconceptions

To identify alternative conceptions, the first step is to interview students or one may use think aloud protocol to explore their thought

process[46]. Five students were randomly chosen from 2<sup>nd</sup> year M.Sc.(Specialization in Theoretical Physics, batch 2012- 2014), Central university of Himachal Pradesh(CUHP). These students have studied the course on ‘Thermodynamics and Statistical Physics’ in their B.Sc. and ‘Statistical Physics’ in M.Sc. 2<sup>nd</sup> semester(two to three months back). Students were known to the interviewer, as interviewer had been a Ph.D. scholar in those days in the same University. Students gave their verbal consent to open up their view in front of interviewer without any hesitation. Various concepts probed in the interview were microstate, macrostate, ensemble, ergodic hypothesis, extensive and intensive variables, equilibrium and entropy. These interviews did not follow any predetermined sequence. Students interviews were audio as well as audio-video recorded, transcribed and analyzed. From these interviews, we directed our research into understanding students difficulties on the concepts of microstate, macrostate and steady state. We have analysed that

- Students were intermixing the concepts of micro-dimensions and micro-state of system, and macro-dimensions and macro-state of a system. Students associate the term micro with ‘less’ or ‘small’ and microstate with ‘less number of particles’, ‘less’ or ‘small size’. They relate the term macro with ‘large’ and macrostate as a state of a system consisting of ‘large number of particles’ or ‘large size’.
- Students actually skip the basic meaning of the word ‘state’ associated with microstate

and macrostate. This shows their lack of understanding in defining the term ‘state’ in the concerned subject.

- They were unable to associate the number of micro-states corresponding to the macrostate of a particular system.

When they were asked “Why Thermodynamic and Statistical Physics is taught as a combined subject”, students responses were :

- In Statistical mechanics, we read microstates to get to know about macrostate.
- Because statistical deals with thermodynamics.
- Because statistics contains little bit of thermodynamics too. We studied thermodynamic laws in studies.
- Some of the properties match.

When asked the question: What do you know about microstate and macrostate? Two students responded saying

- “*I remember, but now I feel that I have forgotten.*”
- “*No idea*”

. It is unexpected, as they have studied these concepts just in their previous semester.

Apart from these formal interviews, we had informal chats/ conversations and discussions on the same topic with other students of the same class. Most of the students’ responses were ‘*I don’t know anything about Thermodynamic and Statistical Physics, madam please don’t take our*

*interview or ask questions*’. These students were either not open to express their thoughts or they might have had a fear in speaking in front of the camera.

To probe students thinking pattern further, open ended questions on the concept of state, system, surrounding, intensive and extensive variable were framed and administered on another group of 22 students of 1<sup>st</sup> year M.Sc.(Specialization in Theoretical Physics, batch 2013- 2015) of the same University. Yet, we focused on one basic open ended question : “**What do you mean by microstate and macrostate of a system?**” Students were requested to avoid answers like ‘*I don’t remember/ I don’t know/ No idea*’. If possible, write your answer with examples. The responses to the open ended question were the part of their tutorial sessions of Statistical Physics class.

Out of 22 students, only 4 students were able to define microstate and macrostate of a system. Rest of them considered microstate and macrostate of a system as ‘*small number of particles/ large number of particles*’. These students were periodically studying Statistical Physics concepts in their regular class. Still, their conceptual understanding is fallacious. Moreover, these are not merely definitions but an important kernel for the whole syllabus of ‘Statistical Physics’. Among 4 students who were able to define macrostate and microstate clearly, only one student was willing to further clarify their response for the written answer. In addition to that we had informal discussions with students of the same class.

To check whether these misconceptions are common among students, unfurnished and yet not validated MCQs were framed and administered on two colleges of Punjab, Jalandhar. Total of 119 students of B.Sc. from HVM and DAV Jalandhar, M.Sc. from DAV Jalandhar and only 8 students from Amity University NOIDA (Uttar Pradesh) participated in this task. MCQs based on microstate and macrostate with percentage of students who chose a particular option are as below in Table I. Although, it was made clear that students should choose only one option in the MCQs, still some students opted for more than one option in MCQs. The results have shown the same pattern which were previously analysed in interviews and open ended questions.

Hence, we have concluded based on our analysis, that students had a vague/ little knowledge regarding the concepts of microstate and macrostate of a system.

### 3.3 Research Questions

The research questions, which have been formulated are as follows :

- Whether students are able to describe and identify thermodynamic state of a system/ macrostate of a system?
- Whether students are able to describe and identify statistical state of a system/ microstate of a system ?
- Whether students are able to distinguish between microstate and macrostate for a given system?

- Whether students are able to build the relationship between the number of microstates belonging to a particular macro-state, for a given system?
- For a given example of a concrete and an abstract system, whether students are able to identify and apply the concepts of probability to the microstates, macrostates and multiplicity of a system?
- Whether students are able to interpret the concept of steady state and thermodynamic limit for a thermodynamic statistical system?

### 3.4 Formulation of Concept Inventory

#### 3.4.1 Content Map of Concept Inventory

The identified alternative conceptions has made it clear that in order to create a concept inventory on microstate and macrostate, one must also create items on microscopic and macroscopic system. This shall clarify students conception regarding the dimensions of the system and the dimensions of the constituents with in the system. Students needs to understand that microscopic and macroscopic system has nothing to do with the state of the system and the state of the constituents with in the system. We have also observed that students find difficulty in defining the system in terms of microstate i.e. in terms of the position and momentum of individual particles present in the system at time  $t$  and in defining macroscopic state of a system, at equilibrium. Again, the concept of multiplicity is utmost important to bind the concept of microstate

Table 1: Showing the percentage of students( $P_s$ ) who chose a particular option in the MCQs

<b>While considering a Thermodynamic system, Microstate of a system may be described as (Option)</b>	<b><math>P_s</math></b>
1. State of a system in which particles have microscopic dimensions.	28.35%
2. State of a system consisting of less number of particles.	33.07%
3. State of a system in which individual position and velocity of particles are specified.	34.65%
<b>While considering a Thermodynamic system, Macrostate of a system may be describe as (Option)</b>	<b><math>P_s</math></b>
1. State of a system in which particles are not of microscopic dimension.	13.39%
2. State of a system containing large number of particles.	59.84%
3. State of a system in which Thermodynamic variables such as P, T or V do not vary with time.	32.28%

and macrostate. The students need to understand that a system will appear to choose a macroscopic configuration that maximizes the number of microstates.[47]. To do so, one should have clear understanding of steady state and thermodynamic limit. Content mapping of the concept inventory is shown in figure 1.

### 3.4.2 Concept inventory Items based on Revised Bloom Taxonomy

Revised Bloom taxonomy(RBT)[48][25] is a  $4 \times 6$  matrix of the knowledge dimension and the cognitive process dimension. The item numbers 1 and 2 are based on factual understanding of microscopic and macroscopic system. The item numbers 3 and 4 are based on the examples of microscopic and macroscopic system. The item numbers 5 and 6 are definition based questions on microstate and macrostate, formulated to identify students conceptual understanding. Item numbers 7, 8 and 9 are application procedural based concrete examples of microstate and macrostate. Item number 11 and 12 are analysed procedural based abstract examples of mi-

crostate and macrostate. Item numbers 10 and 13 are based on evaluative procedural knowledge dimension of RBT. The concept inventory items with RBT knowledge dimension is shown in table 2

### 3.4.3 Items based on the concepts of Microscopic and Macroscopic System

The first four items have been formulated to check whether students are able to define and identify microscopic and macroscopic system. Microscopic means small, of the order of atomic dimensions or less and macroscopic means very large compared to atomic dimension[49]. Description of these items with revised bloom taxonomy (RBT) component and rationale for distractors are shown in table 3

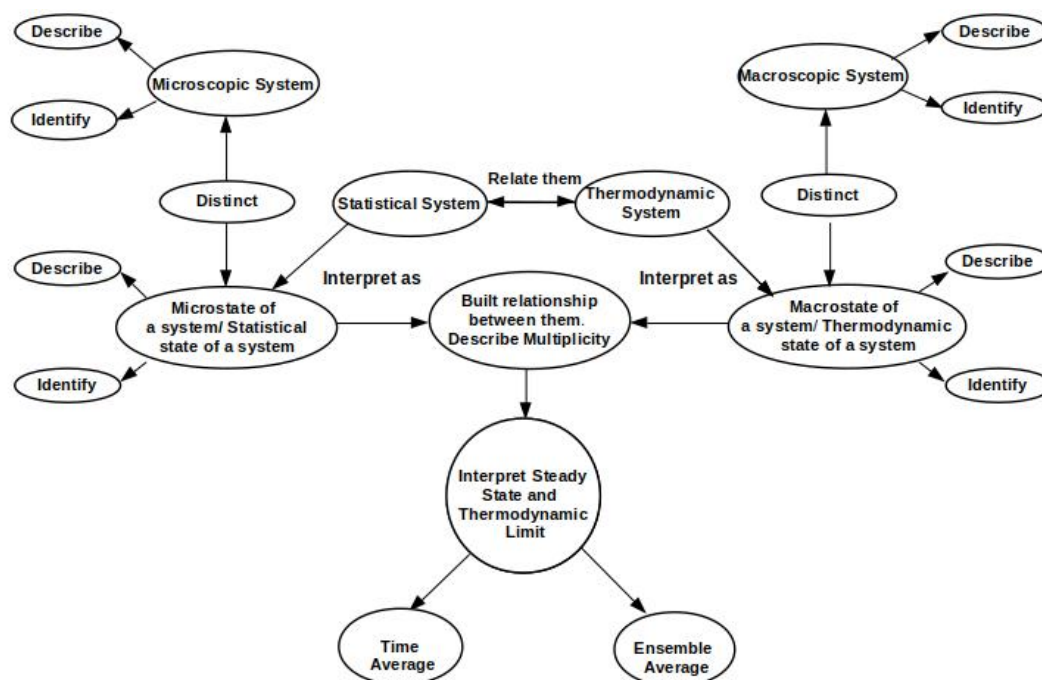


Figure 1: Content Mapping of Concept inventory

### 3.4.4 Items based on the concepts of Microstate, Macrostate, and Steady State of a system

Item numbers 5 and 6 are statement based questions on conceptual understanding of microstate and macrostate. Students find it difficult to define the system in terms of state of all the particles present in the system from statistical point of view and macrostate of a system as a whole, while defining steady state. While administering item numbers 5 and 6, option (a) of an item is a distractor related to microscopic and macroscopic system respectively. While, option (b) corresponds to the definition of microstate and macrostate respectively. The option (c) is both (a) and (b), and is crucial as some of the students mark both (a) and (b) as an answer.

Given the 5<sup>th</sup> item shown in fig 2, administered on 187 students, in a pre-test. 33% students mark option (a) as a response while 25% students mark option (c) as a response. Microscopic state of a system [49] “described in microscopic detail by the most complete specification, according to the laws of mechanics, of all the atoms of the system.” This shows that even though the students have learnt these concepts, they still lack clarity in their conceptual definitions.

Item numbers 7, 8 and 9 are based on procedural application of microstate and macrostate. They have been designed to check whether students are able to identify microstates, macrostates and build the relationship between them. 9<sup>th</sup> item is formulated to access the probability of choosing a microstate associated with

Table 2: The items of Concept Inventory with Revised Bloom's Taxonomy Knowledge Dimension

The Knowledge Dimension	The Cognitive Process Dimension					
	Remember	Understand	Apply	Analyze	Evaluate	Create
Factual		1 and 2	3 and 4			
Conceptual		5 and 6				
Procedural			7, 8 and 9	11 and 12	10 and 13	
Meta-Cognitive						

Table 3: Description of Items of concept inventory based on microscopic and macroscopic system

I.No.	Description of Question(RBT Aspect)	Distractors	Rationale for Distractors
1 & 2	Factual definition of Microscopic and Macroscopic system ( <b>Factual Understanding</b> )	System consisting of few/ large number of particles	Unable to define microscopic and macroscopic system.
3 & 4	Identification of Microscopic and Macroscopic system ( <b>Factual Application</b> )	No Distractor; as all of them are correct responses.	Unable of identify all of them as correct option.

a given macrostate. Item number 10 is designed to incorporate the concept of ensemble average. It represents a system which is not in thermodynamic equilibrium. Further, to ensure that whether students are able to apply and analyse the knowledge of steady state to a new imaginary system, last three items have been devised. These items incorporate a statistical system consisting of  $10^{24}$  particles. The system is being observed after a sufficiently large amount of time. Description of each item with RBT aspect, corresponding distractors and rationale for each distractor are shown in Table 4 and 5.

### 3.4.5 Scoring Key for the Concept Inventory

The scoring key for the concept inventory has been designed in accordance with the conceptual chronology of the items and cognitive process dimensions of revised bloom taxonomy. The conceptual chronology of scoring key means that the items of the concept inventory would be checked in the sequential order of the concepts. The checking of subsequent item within the concepts and among the various concepts depends on the correct response of the previous item(s) [25]. An example of actual post test response of a student is shown in figure 3. Given items are based on

Table 4: Description of items of concept inventory based on the concepts of microstate, macrostate, probable microstates and macrostate, and Steady State

I. No.	Description of Question( <b>RBT Aspect</b> )	Distractors	Rationale for Distractors
5 & 6	Conceptual definition of Microstate and Macrostate of a system ( <b>Conceptual Understanding</b> )	1. State of system consisting of few/large no. of particles. 2. State of system in which particles have microscopic dimension.	Associated alternative conceptions are Microscopic and Macroscopic System. 1. Confused. As they define microstate and macrostate in terms of size and number of particles. 2. Unable to define microstate at an instant t. 3. Unable to define system as a whole, which is in thermodynamic equilibrium.
7, 8 & 9	System of 2-fair coins/ 2-fair dice ( <b>Procedural Application</b> ) 1. Identify the possible microstates/macrostates of a system. 2. How microstates and macrostates are associated with the outcomes. 3. Probability (P) of choosing a microstate associated with a given macrostate if microstates are selected at random.	1. Outcome of a system. 2. $P = \text{Number of microstates associated with a particular macrostate.}$	Unable to identify microstate and macrostate for a concrete system and build the relationship between possible number of microstates belonging to a particular macrostate.
10	Ensemble Average( <b>Procedural Evaluate</b> ) This question focuses on Probability distribution 1. All microstates of a system are equally probable. 2. Microstates corresponding to particular macrostate are equally probable. 3. System will tend towards a particular macrostate which has maximum number of microstates.	All responses are correct(No Distractor).	We would like to see whether students realize and comprehend all the answer to be correct or not. According to Thoads & Roedel [25] choosing more than one correct answer correlates with an increased understanding of the material. Moreover, creating an absurd distractor might mislead them. Our motive is not to confuse or distract them rather to check their conceptual understanding behind the concepts.

Table 5: Description of items of the concept inventory based on the concepts of microstate, macrostate, probable microstates and macrostate, and Steady State

I.No.	Description of Question( <b>RBT Aspect</b> )	Distractors	Rationale for Distractors
11, 12 & 13	<p>A box containing <math>N</math> (non-interacting and independent) <math>10^{24}</math> particles, divided by semi impermeable membrane, into two equal halves. Let <math>n</math> and <math>m</math> denotes the number of particles on the left and right side of the box respectively, such that <math>n + m = N</math>. Particles are randomly moving throughout the box, colliding with each other and with the walls of the box. Some of the particles from the right side of the box enter the left side of the box and vice versa. This process continues for a sufficiently large interval of time and then the newly formed system is observed.</p> <p>1. Identify the microstate(s) of a newly formed system(<b>Procedural Analyze</b>) .</p> <p>2. Identify the macrostate(s) of a newly formed system(<b>Procedural Analyze</b>) .</p> <p>3. Steady State(<b>Procedural Evaluate</b>)</p>	<p>1. Individual states corresponding to each particle of the system could be considered as <b>its all possible micro-states</b> at a particular time <math>t</math>.</p> <p>2. Individual state of each particle on the left side of the box at a particular time <math>t</math> could be considered as <b>some of the micro-states/ one of the possible macrostate</b> of a system.</p> <p>3. Total number of particles on the left side of box is significantly varying.</p>	<p>1. Unable to define system as a whole, at equilibrium.</p> <p>2. Confused to understand whether <b>each particle in a box</b>, considered as its all possible microstate OR <b>state of all particles in a box considered collectively</b> as one of the microstate at a particular time <math>t</math>.</p> <p>3. At equilibrium, microstates of a system are continuously changing but macrostates of a system are approximately constant w.r.t time.</p> <p>4. Unable to interpret steady state in terms of number of particles and time period of observation of the system. (bold words are the key-words)</p>

- 5) Which of the following statement describes a micro-state of a system
- A) State of a system in which the particles have microscopic dimensions.  
 B) State of a system in which individual position and momentum of each particle is specified at a particular time t.
- Choose the correct option:
- a) Only A  
 b) Only B  
 Both A and B  
 d) None of the above

Figure 2: Showing original pre-test response of a student

identifying the possible microstates of a system and in relating the multiple microstates belonging to a particular macrostate of a system. For item number 7 and 8 option (c) and (a) are correct responses respectively. 1 mark shall be given for correct response of an item. As seen in figure 3, student marked (a) as correct option in 7<sup>th</sup> item but answer is option number (c) : Both (a) and (b). Therefore, we count zero mark for item 7. Again, all other subsequent items have been marked zero, as per conceptual chronology based scoring key. This whole process has been followed to minimize random guessing. For items 1, 2, 5, 6; concept inventory has no random check, as they are the first and definition based questions on the concepts of microscopic and macroscopic system, microstate and macrostate of a system. The marking scheme has been discussed in the Table 6.

### 3.5 Validation of Concept Inventory

The validation of concept inventory is based on face and content validity, item analysis and reliability [46].

#### 3.5.1 Face Validity

Face validity is a measure of subjective judgement of the content in terms of examinee [50];

[51]. Students responses are essential to check whether the content or the language of the multiple choice questions (MCQs)/ items are clear to them. The sample size for the administration of three drafts of MCQs in terms of face validity has been shown in Table 7. In each draft, suggestions given by the students have been incorporated.

#### 3.5.2 Content Validity

Three phase content validation process has been adopted. This has been done by examining the contents of topics by the experts/ faculty members, who have been teaching and who had taught the subject ‘Thermodynamic and Statistical Physics’ or ‘Statistical Physics’, at undergraduate or postgraduate level [46]. In each phase, different set of experts have been chosen. For each item, three point rating scale i.e. Good, Average or Poor has been designed. Experts gave their valuable remarks and suggestions in terms of :

- content relevance (the content of the question is relevant to the topics)
- content clarity (the content being asked in the question is clear to the respondent)

Let us consider a system of 2 fair 6 faced dice, such that, the probability of obtaining each of the outcomes is equal. By tossing the dice simultaneously a large number of times and noting the individual outcomes of each of the dice every time, results in various possible micro-states associated with the system.

एक ऐसे system की कल्पना कीजिए जिसमें 2 fair 6 faced dice हों जिससे कि सभी outcomes के मिलने की probability equal हो. बार-बार 2 dice को एक साथ उछालने पर मिलने वाले परिणाम, system से जुड़ी micro-states को दर्शाते हैं.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$H, T$

$(1, 4) (4, 1) =$

Figure 1: Various possible micro-states on tossing a system of 2 fair 6 faced dice simultaneously

$(1, 6), (6, 5), (5, 4), (4, 3), (3, 2), (2, 1)$   
 $(6, 1)$

Following questions (7, 8 and 9) are based on the given system.

- 7) What would you conclude about the micro-state and macro-state associated with the outcome in which total of two dice comes out to be five ?
  - A) Micro-states are (1,4), (2,3), (3,2), (4,1)
  - B) There are four micro-states in a given macro-state.

Codes:  
 [a] Only A  
 b) Only B  
 c) Both A and B  
 d) None of the above
- 8) Outcome with the greatest multiplicity is/are the  
 (Number of micro-states belonging to a particular macro-state is known as multiplicity)
  - a) State with sum 7
  - b) State with sum 12
  - c) Both a and b

Figure 3: Showing original post-test response of a student

- language clarity (the language clearly conveys what is being asked in the question)

Suggestions given by the experts in the third phase of content validation process are as below: According to one of our experts, “Q’s have been designed for the purpose of bringing out the level of conceptual understanding of a basic problem in statistical mechanics- the meaning of microstates and its correlation with the thermodynamic probability of the corresponding macrostate of a thermodynamic system”. “Thermodynamic probability of a system is defined as

the number of microstates associated with the system. It is not normalized. The system has no choice, it can remain in any microstate with probability which is equal to the reciprocal of number of microstate.”. Expert suggested us to change the options of item number 9. This suggestion has been taken care of while modifying item number 9. Therefore, in formulating items, the concepts of probability and thermodynamic probability must be taken care off. This may lead to the confusion among students. Same expert suggested us to create short objective questions

Table 6: Scoring key of concept inventory based on conceptual chronology of topics

Topic	Item Number (I.No.)	Answer (A) : 1 mark for correct answer
Microscopic and Macroscopic System	1 and 2	<b>c</b> (Definition based independent questions on <i>microscopic and macroscopic system</i> )
	3	<b>d</b> (Identification based <i>microscopic system</i> , If I.No. 1 is incorrect then I.No. 3 is incorrect too)
	4	<b>d</b> (Identification based <i>macroscopic system</i> , If I.No. 2 is incorrect then I.No. 4 is incorrect too)
Microstate, Macrostate and Steady State of a system	5(a), 5(b) and 6	<b>b</b> (Definition based independent questions on <i>microstate and macrostate of a system</i> )
	7	<b>b [for 7(a)], c [for 7(b)]</b> (either I.No. 5 or 6 is incorrect then I.No. 7 is incorrect too)
	8	<b>c [for 8(a)], a [for 8(b)]</b> (If I.No. 7 is incorrect then I.No. 8 is incorrect too)
	9	<b>d</b> (If I.No. 8 is incorrect then I.No. 9 is incorrect too)
	10	<b>d</b> ( <i>Based on Probable microstates and macrostates</i> ; If 9 is incorrect then I.No. 10 is incorrect too)
	11	<b>a</b> (If either I.No. 11 or 12 is incorrect then both are incorrect. Moreover, if either I.No. 10 is incorrect then 11 and 12 are incorrect too)
	12	<b>b (0.5 mark)/ d(1 mark)</b>
13	<b>d</b> (If either I.No. 11 or 12 is incorrect then I.No. 13 is incorrect too)	

and MCQ items using the examples of atomic systems, placed in a magnetic field, with total angular quantum number  $j = 1/2$  and  $j = 5/2$ . Researchers shall try to create the same later on while formulating the modified version of the concept inventory.

Another group of three experts suggested to us that “Statistical Mechanics deals with large and here(item number 13) fluctuation of thermodynamic parameters for system in thermodynamic equilibrium is practically nil; in that case there is no experimental difference between option b & c. So, correct option should be b & c.

There is a difference between physics and mathematics!”

Here, option b and c were

- Total number of particles on the left side of the box is approximately equal to  $N/2$ .
- Total number of particles on the left side of the box is exactly equal to  $N/2$ .

Suggestion have been taken care of while modifying the answer of item number 13. Again, same group of experts pointed out that for items 11 and 12, the words partial/ fractional states

Table 7: The face validation of the concept inventory in terms of concept and language clarity for various draft is shown

	Name of the Institute	Sample
1 <sup>st</sup> Draft (Try Out - 1)	H M V, Jalandhar, Punjab	41 students (B.Sc. 2 <sup>nd</sup> year, 2013)
	D A V College, Jalandhar, Punjab	35 students (B.Sc. 2 <sup>nd</sup> year, 2013), 43 students (M.Sc. Physics 1 <sup>st</sup> and 2 <sup>nd</sup> year, 2013)
	Amity University, Noida, U.P.	8 students (M.Sc. Physics, 2013)
2 <sup>nd</sup> Draft (Try Out - 2)	CUHP, Dharamshala, H.P.	25 students (M.Sc. Physics 2 <sup>nd</sup> year, Batch 2013-2015)
3 <sup>rd</sup> Draft (Try Out - 3)	CUHP, Dharamshala, H.P.	25 students (M.Sc. Physics 2 <sup>nd</sup> year, Batch 2014-2016), 5 students (Persuing P.hD. in Physics, Batch 2015-2018)

have been used. They wrote “We do not understand the concept of partial/ fractional states.”. We removed the words : partial and fractional states and the language of these item numbers has been revised and simplified.

One of our experts has suggested to test the knowledge of ensemble using the concept of phase space. To do so, we need to create items on configuration space and phase space. We shall try to create them in our modified version of concept inventory.

To a large extent, suggestions of various experts have been incorporated. It should be noted that some of the items were marked poor in terms of language clarity. Therefore, researchers have simplified and rectified those questions and further converted them into bilingual, English as well as native language(Hindi).

The final version of concept inventory con-

tains 18 items. It is further divided into two equivalent tests, pre test and post test items. Item numbers 3, 4, 5, 7 and 8 are separate for both tests. Experts in all three phases have checked their equivalence in terms of content relevance and difficulty level. Therefore, we have a total of 13 items in each test and any of them could be used as pre-test or post-test.

### 3.5.3 Item Analysis : Item difficulty index(P) and Item discrimination index(D)

Item analysis includes item difficulty and item discrimination index. The same has been calculated for concept inventory post test results for the main study.

According to Maloley et.al [22], item analysis should be conducted on post-test scores

Table 8: Description of sample size(N=121 students) for Item analysis is shown

Name of the Institute	Sample
GPGC, Una, H.P.	N <sub>1</sub> = 59 students (B.Sc. V Semester Major Physics)
CUHP, Dharamshala Regional Workshop on PER	N <sub>2</sub> = 38 students
‘Learning By Doing’	SSU, Palampur, H.P. (9 students of M.Sc. Physics 1 <sup>st</sup> year)
	DAV, Kangra, H.P. (15 students of M.Sc. Physics 1 <sup>st</sup> year)
	DCE, Rait, H.P. (15 students of B.Ed. with Teaching of Science)
CUHP, Dharamshala, H.P.	N <sub>3</sub> = 24 students (M.Sc. Physics 1 <sup>st</sup> year)

because pre-test scores lacks common sense ideas and unfamiliarity with the common terms. Moreover, the concept inventory has been created keeping in mind that students have misconceptions/ alternative conceptions on the chosen concept. Therefore, item difficulty and item discrimination of pretest items is meaningless. We could study the same only after administering an intervention. Researchers have prepared and administered set of modules and an intervention [45] whose finding will be discussed in a subsequent research paper. The description of main study sample size for item analysis is shown in table 8. Main study data constitute those items which were based on the suggestions given by the experts till the second phase of content validation and third drafts of face validity.

To calculate [52] the item difficulty index and item discrimination index of each of the items in the concept inventory, the test scores of the students were arranged in descending order w.r.t. to students' response to the particular item. The top 27% scores of the data for a particular item represents the upper group. The bottom 27% scores of the data represent lower group. The formula for

calculating P and D are as follow :  $P = \frac{(RU+RL)}{(NU+NL)}$ ;  
 $D = \frac{(RU-RL)}{(NU)}$

where RU and RL are the total number of students who responded correctly in the upper and lower groups respectively. NU and NL are the total number of students in the upper and lower groups respectively. The values of P and D calculated from the post test conceptual chronology scores are shown in Table 9. A difficulty value of 0.5 is usually taken as an ideal value [22]. Item numbers 1 to 10 fall under or near ideal item difficulty. For 11<sup>th</sup>, 12<sup>th</sup> and 13<sup>th</sup> item, the difficulty index has quite a low value. This could be possibly because these items belongs to higher cognitive dimensions of RBT. These items have been already pointed out to be bit difficult by the experts in the third phase of content validation. The same have been rectified as discussed in content validation section. The modified items had to be retained, as they illustrate relationship between microstates, macrostates and steady state. A discrimination index 0.40 and above are considered as very good items. Value between 0.30 to 0.39 are considered as reasonably good but possibly subject

to improvement [52] (pp. 232). All other items except 1, 12 and 13 fall under very good items. It has been pointed out [24] that, according to authors [46], an item which is important from faculty point of view and at the same time shows a significant gain in the post-test results, needs to be retained in the concept inventory. Hence, every item is important as they have already been validated by the experts. The only solution is one needs to improve the methodology of teaching these concepts to bring about better conceptual clarity among students.

### 3.5.4 Reliability

Reliability is the level of consistency over time. It is to check, as to what extent, whether the tests yield the same results on repeated trials. Reliability has been calculated using test-retest method, to check for stability over time. The formula for calculating reliability using Karl Pearson coefficient of correlation is

$$r = \frac{\sum_{i=1}^n ((x_i - X)(y_i - Y))}{\sqrt{\sum_{i=1}^n (x_i - X)^2 \sum_{i=1}^n (y_i - Y)^2}}$$

Where n = Sample size

$x_i$  = Sum of the correct responses of students to test item i for a time period say  $t_1$ .

$y_i$  = Sum of the correct responses of students to test item i for a time period say  $t_2$ .

X = Mean of variable x.

Y = Mean of variable y.

For one set, say pre-test items, have been administered on B.Sc. V Semester, Major Physics, MPGC, Amb, H.P. (n = 66 students) at two different times. The reliability coefficient for pre-test set is found to be 0.43. The low coefficient value may be due to students misconceptions or tendency to make guesses. Moreover, in test-

retest method it is not possible to apply conceptual chronology of scoring key.

The inter reliability of the post-test of main study (N=121 students) has been calculated using Kuder Richardson 20 formula as

$$R = \frac{N}{(N-1)} \left( 1 - \frac{(\sum_{i=1}^N (p_i q_i))}{(\sigma^2)} \right)$$

where variance =  $\sigma^2 = \frac{\sum_{i=1}^n (x_i - X)^2}{n}$

n = Sample size

N = Number of items

$p_i$  = Sum of correct response of students to test item i.

$q_i$  = Sum of incorrect response of students to test item i.

$p_i + q_i = 1$

$x_i$  = Sum of scores of a student i.

X = Mean of variable x.

The value of R comes out to be **0.87** which is high [22] and falls in the acceptable range [52], (pp. 86).

## Conclusion

The concept inventory on Microstate, Macrostate and Steady State has been developed at a level suitable for students taking undergraduate statistical physics course. These efforts are based on the inputs received through a process of elaborated discussions with experts and identified misconceptions based on the analysis of students' responses to the interview questions, open ended questions and multiple choice questions. It has been validated via face and content validity, item analysis and reliability. The conceptual chronology of scoring key has been applied to minimize random guessing.

Table 9: Calculated value of Item Difficulty(P) and Item Discrimination(D) of Post test concept inventory on N = 121 students.

I.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
<b>P-value</b>	0.77	0.48	0.56	0.41	0.71	0.64	0.50	0.50	0.50	0.48	<b>0.29</b>	<b>0.20</b>	<b>0.18</b>
<b>D-value</b>	0.39	0.67	0.52	0.64	0.58	0.73	1.00	1.00	1.00	0.91	0.58	0.39	0.36

#### 4 Suggestions for further research

We intend to create more items for the concept inventory based on suggestions received from various experts. One of our experts has suggested to create short objective questions useful for testing smaller sample size. One could also create items of concept inventories based on microcanonical, canonical and grandcanonical ensemble. These concepts shall further strengthen the basic ideas of microstates and macrostates. One could formulate strategies and test this concept inventory at different institutions across different levels. We found that there is a need to design a constructivist based approach towards teaching of theoretical framework of statistical physics ideas. For this, we must adopt a model in which students, teachers, environment and content interact strongly with each other[53].

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## Outlook for ET Intelligence Lookout

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### Abstract

The idea of existence of extra-terrestrial intelligence is intriguing. Our universe is about 13.8 billion years old; the *Akaash Ganga* (our galaxy, the Milky Way) is about 13.6 billion years old [1]. By a well-considered estimate, the number of habitable planets in our galaxy alone is approximately 300 million [2]. It is therefore counter-intuitive to suspect that our civilization is the only one in the entire universe. On the other hand, no evidence of alien life has yet been found. This dilemma is often referred to as the 'Fermi Paradox', after Enrico Fermi. The Italian- American physicist Fermi is best known for his contribution to the *Manhattan project*; for the *Chicago Pile-1*, which was the world's first nuclear reactor. Fermi never published any work related to the paradox named after him, and the conundrum had troubled man long before Fermi. Fermi had however raised disquiet about it in a casual chat with his colleagues which resulted in his name getting associated with it. The Fermi paradox is the contradiction between the scale of the universe and the lack of evidence for intelligence elsewhere in the cosmos [3]. In this article, we attempt a briefly overview of the trepidation cause by the Fermi paradox.

## 1. Drake Equation

The American astronomer Frank Drake developed an equation known after him in the backdrop of the *Project Ozma* which was one of the earliest initiatives for the Search for Extra-Terrestrial Intelligence (SETI) using interstellar radio waves [4]. Drake's project came up in the backdrop of an article published in 'Nature' in 1959, titled *Searching for Interstellar Communications* by physicists Giuseppe Cocconi and Philip Morrison [5] who argued that radio technology had sufficiently advanced and sensitive to pick up interstellar radio messages. Cocconi and Morrison further suggested that these messages might have been transmitted at 1420.4 MHz frequency or 21 cm wavelength of the photon in the region of the electromagnetic radiation below the microwave segment, resulting from the spin-flip transition in the hyperfine levels of the neutral Hydrogen atom [6,7]. Being the most abundant species in the physical universe, it would hence be the natural choice of a developed intelligent form of life to be used for interstellar communication. The occurrence of this transition itself is very rare - only once every 10 million years - the high abundance (~75%) of neutral hydrogen in the universe would make this transition observable. The Nature article prompted Frank Drake to setup a systematic plan to look for interstellar signals in his *Project Ozma*. He used a 26 m dish of the National Radio Astronomy Observatory, Greenbank, now called the Greenbank Observatory, to look out for such signals from two nearby stars, 'Epsilon Eridani' and 'Tau Ceti', for 6 hours a day between April 1960 and July 1960. This exploration turned out to be unsuccessful, but led Drake to host a meeting in 1961 at the Greenbank Facility at which the invitees included prominent physicists and astronomers including Carl Sagan, Philip Morrison, John C. Lily etc. Drake formulated a probabilistic model to estimate the chance that extra-terrestrial intelligence exists. Essentially, he developed a quantitative guesstimate of this probability as an equation, now called the *Drake equation*. It set up the agenda for the SETI

(Search for Extra Terrestrial Intelligence) meeting.

It is a formula to reckon the number of communicative civilizations present in our galaxy who could produce detectable electromagnetic emissions. The Drake equation is:

$$N = R_* \times f_p \times n_e \times f_l \times f_i \times f_c \times L$$

where

$N$  = the number of communicative civilizations in our galaxy,

$R_*$  = the average star formation rate in our galaxy,

$f_p$  = the fraction of those stars that have planet,

$n_e$  = the average number of planets that are potentially habitable per star,

$f_l$  = the fraction of those planets on which life eventually appears,

$f_i$  = the fraction of those planets on which intelligent life emerges,

$f_c$  = the fraction of those civilization that develops the technology to release the detectable signals in the space, and

$L$  = the average length of the time such civilization release signals.

Most of the terms in the Drake formula are conjectures. When Drake developed this relation, only formation rate of stars was known. Techniques like gravitational microlensing [6] and missions like Kepler have enabled us to make a decent prediction of  $f_p$  and  $n_e$ . The remaining four terms remain elusive. Statistical calibration is difficult, our Earth being the only known habitable planet.

The last term  $L$  may have the greatest impact on the outcome, but there is no mechanism known to determine it. The guesses range from 10 to 10 million, Drake's original estimate of  $L$  was 1000 to 100 million years [7]. It is common to consider pessimist and optimistic values of  $L$  from 304.5 years to 1 billion years, as suggested by Michael Shermer [8] and David Grinspoon [9]. Michael Shermer arrived at this figure by calculating the average length of 'modern' civilization following the Roman empire, taking into account 28

different civilizations that can be considered to have developed since the dawn of the humanity. David Grinspoon proposed that if a society overcomes its self-destructive propensity, it can live for billions of years. As one can expect, the value one gets for the number of communicating civilization from the Drake equation varies vastly. Current estimate ranges from about  $9.1 \times 10^{-13}$  to 15,600,000. The former employs  $R_* = 1.5$  to  $3 \text{ y}^{-1}$  [10],  $f_p \times n_e \times f_i = 10^{-5}$  [11],  $f_i = 10^{-9}$  [12],  $f_c = 0.2$  [7] and  $L = 304.5$  [8], while the latter uses  $R_* = 1.5$  to  $3 \text{ y}^{-1}$  [10],  $f_p = 1$  [13],  $n_e = 0.2$  [14],  $f_i = 0.13$  [15],  $f_i = 1$  [16],  $f_c = 0.2$  [7],  $L = 109$  [9]. The huge difference between the results raises concern about the utility of the Drake equation, but the significance of Drake's work stems from the fact that it provides a starting point for corrections and improvement as may be required. The Drake equation has been modified by other astronomers, either by adding to or modifying its factors. In his famous paper "*The Great Silence – the Controversy Concerning Extra-terrestrial Intelligent Life*," David Brin [17] proposed introducing a component linked to contact cross-section ( $C$ ) to account for the effect of colonisation. Alexander Zaitsev [18] has advocated including a "METI" (Messaging to Extra-Terrestrial Intelligence) component, which accounts for the proportion of civilizations that can purposefully interact. Based on the Drake equation, astronomer Sara Seeger [19] has suggested a new equation that looks for indications of life by looking for planets that emit biosignature gases. John Gertz [20] recently altered the Drake equation by substituting the rate of star formation with the number of stars in our field of view ( $N_s$ ),  $n_e$  with the number of bodies possibly livable on or under their surfaces ( $n_{th}$ ) and  $f_c$  with the parameter  $f_d$  which accounts for overall detectability of a civilization. Karl-Florian Platt [21] presented a simplified Drake equation that only considers the number of planets capable of supporting life. He also added a change to the simplified Drake equation to account for habitable moons. It remains to be

established if these modifications constitute a significant advance.

## 2. Fermi Paradox

Michael H. Hart must be credited for resuscitating Fermi's query as he was the first one who published a detailed work in 1975 [22] around the Fermi's original poser. His study was essential in hastening the discussion among scientists over the presence of extra-terrestrial intelligence. Historically, there have been two camps: "Contact Optimists" and supporters of the "Uniqueness Hypothesis", referred to by David Brin [17]. The first group includes Frank Drake, Carl Sagan [23][24], Bernard Oliver [25], and Philip Morrison [5], while the second includes Michael H. Hart, Leonardo Ornstein [26], and Frank J. Tipler [27], amongst the prominent ones. Interestingly, most physicists were contact optimists whereas most biologist favored the uniqueness theory. The uniqueness hypothesis is the most common answer to the Fermi Paradox. It denies the existence of extra-terrestrial civilizations in our galaxy and claims that civilization on the earth is the only civilization at least in entire galaxy. Common arguments for these two posturing platforms are:

*Rare Earth Hypothesis:* It states that the emergence of sophisticated multicellular life is uncommon; special and fortuitous circumstances are required for intelligent life to develop. These include a planet's location in the galactic as well as in continuous habitable zone, the presence of a large moon, the presence of a giant planet (like Jupiter in the planetary system), the availability of a magnetosphere, plate tectonics, lithosphere, atmosphere, oceans, and other known and unknown events which bred evolution from simple prokaryotic cells to intelligent life on Earth. In their book 'Rare Earth: Why Complex Life is Uncommon in the Universe', Peter D. Ward and Donald Brownlee explain this in detail [28]. A rip-off of the Drake equation is also proposed, termed as the "Rare- Earth Equation".

The criticism for this hypothesis comes in the form of Principal of Mediocrity.

*Principal of Mediocrity*: This is an extension of Copernicus principle, which assumes that the Earth is not a special planet in any sense. Both Sagan and Drake have advocated this. The discovery of exoplanets [29] gives credence to the view that there could be a large number of planets in the habitable zone. Furthermore, the origin of extra-terrestrial life cannot be necessarily limited to an earth-like environment. For example, scientists are working on approaches to look for signs of "second genesis" in worldly oceans [30]. There exists a variation of the "Rare-Earth Hypothesis" that considers the origin of extra-terrestrial life a not-so-common thing but assumes intelligence to be rare. Many biologists like Ernst Mayr [12], subscribe to this hypothesis. This hypothesis however suffers from a fundamental flaw: It is difficult to test. The Rare earth hypothesis can be tested by looking at bio-signatures but testing intelligence is a tedious task.

An interesting hypothesis was advanced by economist Mark Hanson in his online essay – "The Great Filter – Are We Almost Past It?" in 1996, which was later updated in 1998 [31]. Hanson listed vital steps for a civilization to develop. In the nine steps of evolution he proposed, the first was "the right star system" and the last was "Colonization explosion." He claimed that extra-terrestrial intelligence is rare as several of these steps may be unattainable. Our civilization is already possibly in the eighth stage - "*a civilization advancing toward the potential for a colonization explosion*" making the final stage "colonization explosion" unattainable by us [31].

The second approach mainly works on either disproving resolutions proposed by the optimists or using the Drake equation itself to find an ignorable number of communicative civilizations making the Earth the lone civilization. Hart [22] presented counter arguments against major explanations of the Fermi paradox offered by the optimists. He stated that space exploration was

not only achievable but also practical. He considered a speed of starships to be one-tenth the speed of light and presumed no time lag between production of a new spaceship and arrival of the previous one.

He predicted that the entire galaxy would be covered in 650,000 years. He rejected *sociological explanations* because they were devoid of a provision to test their validity. He thought that a *temporal explanation* that extra-terrestrial beings did not have sufficient time to reach our civilization was although theoretically conceivable but considered it highly unlikely as for it we had to go by the fact that our species evolved less than one time unit (1 time unit =  $2 \times 10^6$  years) after the first civilization capable of interstellar communication, which took at least 5000 time units to emerge. He asserted that a great degree of coincidence was required for this to occur, therefore the explanation is not very plausible.

### 2.1 Von-Neumann Probe

Tipler in his paper "Extra-terrestrial intelligent beings do not exist" [27] argued that any civilization would attempt to explore and colonize the galaxy and it can be done with self-replicating space probes, known as *Von-Neumann Probes*.

A *Von-Neumann Probe* is a potential self-replicating spacecraft that could be utilized for the exploration of space. This machine may duplicate itself utilizing raw material from a distant star-system and thereby visit a whole galaxy [32]. It is named after the Hungarian-American scientist John Von Neumann, who was the first one to propose a mathematical framework for the construction of self-replicating machines. He explored the essential design of a Universal Constructor in his book *Theory of Self-Reproducing Automata*. After John von Neumann's death this work was concluded by Arthur W. Burks in 1966 [33]. His goal was to create a machine whose complexity would increase naturally through natural selection, similar to biological evolution. Chris Boyce

advocated for its usefulness in the context of galactic colonization in his book *Extraterrestrial Encounter: A Personal Perspective* [34]. Robert Freitas [35] gave an early sketch of such self-replicating probes in 1980. The discussion of the Von-Neumann probe finds utility since current advances in AI and Machine Learning bring us one step closer to developing such probes practically.

Development of self-replicating space probes is contingent upon the enhanced understanding of the extra-terrestrial signals. These probes can be of different types, depending on their proposed applications. A probe could also possibly adapt like *transformers*. This pre-supposes in-built capabilities to transform depending upon understanding of the deep space signals through AI & ML (Artificial Intelligence and Machine Learning) processing. Other types of space probes would make use of available resources of the immediate environment and evolve into entirely new exploratory robots. Huge amount of data would be required to make such machines capable of learning and facilitate their transformations. The outcomes can greatly depend on the size of the payload.

Recent advances in AI & ML can offer significant insights in the functioning of Von-Neumann probes. Von Neumann Self-Replicating Space Probes can only benefit by integrating AI and ML based algorithms and signal processing. Similar to the prevailing explorations of deep space by using satellites and deep space signal systems, the AI and ML integrated signal processing systems can offer enhanced understanding of the deep space. The key challenge, in this context will be to train the machines to learn the signals that the probes of exploratory instruments might encounter, for which *a priori* learning process seems not possible on account of the lack of clarity and non-availability of information to train and discern the signals. In this context, available signals need to be studied and used to generate a basis to detect other signals that emanate from unusual extra-terrestrial activities. This process would need to

continue iteratively to equip the machine to develop classificatory processes. The outcomes would be supportive to the probes of the future.

While the AI and ML has the potential to enable our understanding when used in probes, the prevalence of the probe as such has been debated. Tipler argued that neither any sign of such probes has been observed, nor does the Drake equation yield a satisfactory value. If we assume that the upper limit for the probability of intelligent life that eventually attempts interstellar communication will evolve is  $1/N$  where  $N$  is the number of stellar systems that are older than 5 billion years and consider  $N$  to be around 1011 Tipler argued that our civilization seems to be the only one. He further argued that the universe is  $\sim 13.8$  billion years old, any previous civilization that has existed would have made a voyage to space, eventually colonizing the galaxy, and leaving their mark.

The ‘Hart-Tipler Conjecture’ was criticized by Carl Sagan and William Newman in their response *Galactic civilizations: Population dynamics and interstellar diffusion* [36]. To understand interstellar colonization, Sagan and Newman employed a model similar to one that is used by biologists to study animal populations. They found Hart’s result to be unrealistic, and expected the expansion to be much slower. Also, they questioned Hart’s logic that a civilisation will always tend to expand, owing to the fact that it is impossible to foretell how an alien society might think.

### 3. Solution to Fermi Paradox

Stephen Webb in his book *If the Universe is Teeming with Aliens ... Where is Everybody?* [37] has listed 75 solutions to the Fermi paradox.

A few important explanations of the Fermi Paradox are listed below (They do not necessarily come from the book)-

#### 3.1 Extinction:

An alien civilization may have existed at some point but was wiped out by periodic extinction, similar to dinosaurs on Earth. Possible causes include runaway heating, runaway cooling [38], and cosmic gamma ray bursts [39]. At this juncture, AI and ML based self-replicating probes perhaps may be of support to understand such evolutionary processes in case the relevant data forms part of the probe.

### 3.2 The First-being Hypothesis:

Earth could be the first technologically equipped civilization. Other civilizations either are not yet born, or not yet sufficiently technologically advanced. Given the age of the universe, this explanation seems to be implausible to some scholars [39].

### 3.3 Self -destructing tendency of intelligent life:

When intelligent life becomes technologically advanced enough, it would engage in self-destructive processes such as nuclear war, bioterror, genetic deterioration, environmental degradation, anthropogenic climate change etc. Badly developed artificial intelligence, natural disasters etc. are all potential reasons for a civilization's demise, resulting from technological progress [40].

### 3.4 Brief Window Hypothesis:

This postulates that an intelligent civilization might exist only for a brief window of time and then it dies out [41]. In his work *The Search for Signals from Other Civilizations*, Von Herber Sebastian [42] suggested that a civilization's existential period might be shorter than the time necessary to establish connections with others. He argued that advances in science and technology occurred mostly from

(a) ambition for supremacy

(b) from a desire for comfortable existence.

However, both of these can trigger destructive compulsions. Adam Frank investigated the implications of anthropogenic climate-change in his paper *The Anthropocene Generalized: Evolution of Exo- Civilizations and Their Planetary Feedback* [43]. From his studies on extinct civilizations such as Easter Island (Rapa Nui) he concluded that anthropogenic climate-change leads to failure of a civilization. According to Jacob D. Haqq-Misra and Seth D. Baum's study *The Sustainability Solution to the Fermi Paradox* [44], an exponential expansion of civilization is not sustainable on a galactical level and is prone to collapse. Majority of theories under this umbrella are however not free from circular logic. In order to enable our understanding, a ML model that make use of the anthropogenic data and the terrain, perhaps can supporting simulating models.

### 3.5 Berserker Hypothesis:

This hypothesis emerged against the backdrop of *the Von-Neumann probes*. "Berserker" is the famous series of science-fiction novels written by Fred Saberhagen. The central narrative centers on self-replicating robots that aim to annihilate all life. The hypothesis wonders that if a civilization became advanced, made self-replicating probes to explore its galaxy, and then these probes went rogue, destroying whoever came in their way, no sign of any civilization would be left. Tipler advocated the use of self-replicating probes in his 1981 paper and went on to claim our civilization to be the sole civilization on the basis of no-detection of any such probes [45]. Sagan and Newman [46] saw such probes as harmful because they may corrupt and destroy the parent civilization. In his paper *The Great Silence – the Controversy Concerning Extra-terrestrial Intelligent Life*, David Brin also mentioned that if the parent civilization sends these self-replicating probes to other stars and one of these probes turns out to be xenophobic, then it can create further

copies of itself and eventually create a whole army of xenophobic probes. Edward Harrison [47] came up with modification of this idea, saying that if a civilization overcomes its self-destruction propensity, it can even build lethal probes as a safety mechanism to defend its civilization from other civilizations.

The critique of this concept stems from the fact that even if lethal probes destroy an extra-terrestrial civilization when they come into contact, they are expected to leave trace behind. Anders Sandberg and Stuart Armstrong [48] discovered in their paper *Hunters in the Dark: game theory analysis of the deadly probes scenario* that even with a slow expansion rate of these probes, they could have spread out across the galaxy and contacted mankind.

Duncan Forgan of the University of St. Andrews Centre for Exoplanets Science in his recent study [49] argued that even if berserker space probe attacks the extra-terrestrial civilization it won't be able to stop them from spreading throughout the galaxy.

### 3.6 Aliens are different from us:

Alien species might have a very different cognitive capability than us. Carl Sagan speculated that their thought patterns might be vastly different from ours [50]. Their communication technology and the frequencies on which it operates might be entirely unusual for us. Their mathematics can be different. Although we may assume the outcome of physical laws to be valid regardless of the convention used, it is feasible that they have evolved to the point where they have moved beyond radio to other regions of the spectrum or even beyond electromagnetic radiation signals. According to Seth Shostak's article [51], as a civilization advances, communication technology advances as well, and hence radio leakage would decrease significantly. Even if extra-terrestrial species send their signals through electro-magnetic radiation, the

frequencies adopted by them can be strange enough that we may dismiss the same as noise.

In 2004, American astronomers Jay M. Pasachoff and Marc L. Kutner suggested interstellar communication through neutrinos in their article in the *Cosmic Search* magazine [52]. Neutrinos are elusive, travel at nearly the speed of light and are broadband, making them a good candidate for interstellar communication. A group of physicists headed by Arjun Berera at Edinburg University has conjectured that quantum communication would be an effective medium [53]. However, we can never be certain about which method extra-terrestrial species would use, based on where their civilization falls on the *Kardashev Scale*.

The Kardashev Scale is a hypothetical measure to determine the level of technological advancement of a civilization. Originally, it was proposed by Soviet astronomer Nikolai Kardashev in his paper *Transmission of information by Extra-terrestrial Civilizations* [54]. He divided civilization into three types based on their energy consumption. Type I is a civilization that can gather and store all of the energy available on its planet; Type II is a society that can directly harness the energy from a star; and Type III is a one that can collect all the energy in the whole galaxy. The scale has been a subject of re-evaluation and modification. However, the wonders never stop in this field. A recent newspaper report, [69] highlighted occurrence of a powerful radio signal from the edge of the universe. There was observance of an eight billion years old pulse that has been traveling for more than half the lifetime of the universe. The fast radio burst (FRB) was reported as intense, brief flashes of radiation that are invisible to the naked eye but was detected by radio telescopes. This offers the lead for studying the "cosmic web" of matter between galaxies. The report showed that, the degree to which bursts slow down correlates with the distance it travelled. More distant and extreme fast radio bursts promise to reveal further secrets about the universe. As and when more advanced telescopes

are introduced, more galaxies could be observed, paving the way for resolution to many a astronomical mysteries.

### 3.7 Sociological Explanations:

A typical approach in this category argues that alien species may not even desire to colonize the galaxy, or may prefer to stay alone. Some have even theorized that aliens may have understood the riddles of the world and may be no longer explore anything further [55]. Another point of view is that aliens may have quit living on planets and may have become very selective in their interactions with other civilizations [17].

*The Zoo hypothesis* is the most significant theory under this umbrella. It is that Aliens have already discovered our civilization, but have opted not to disturb us by revealing their presence. This idea, proposed by Allen Ball in 1973 draws a parallel between animals imprisoned in zoos and wildlife sanctuaries in their natural environment and the human civilization [56]. Aliens may be watching us the same way that we watch animals in a zoo. They are technologically advanced enough to hide from us and will expose themselves only when we have evolved sufficiently in terms of politics, ethics, and technology. One variant of this concept is the laboratory hypothesis, which holds that the whole Earth is a giant laboratory where humans are exposed to extra-terrestrial experimentation. It is however criticized for its speculative and not-testable nature. Besides, it would take only one rebel among the aliens to break this code of conduct and reveal the aliens; this hasn't happened.

### 3.8 Planetarium Hypothesis:

This is a variant of the Zoo hypothesis. It is based on the theory proposed by Stephen Baxter that our world is virtual [57]. We find the universe empty because it is programmed in that way. All sociological explanations are speculative, and cannot be tested.

### 3.9 Economic Explanations:

If interstellar travel is assumed to be technologically feasible for any civilization, the resources required for galaxy colonization will be prohibitively expensive. Hence, there is a very remote possibility for any civilization to expand in the whole galaxy. In his paper, Landis suggested a model in 1989 for space colonization based on *percolation theory* [58]. Landis chose two parameters,  $P$ , the probability of development of a colonizing civilization and  $N$ , the connectivity of the graph, and concluded that either the probability of space colonization is less than a threshold limit or the Earth lies in empty voids of the galaxy which are not visited by any civilization. In the latter case, he argued Earth might be in an un-colonized void, surrounded by non-colonizing civilizations. Keith B. Wiley published a paper in 2012 [59] in which he added two extra parameters to Landis' original model:  $K$ , the lifespan of a colony, and  $M$ , the mutation rate in the colonizing state of a colony. He anticipated that by taking these two characteristics into account, the entire galaxy will be colonized. Sending information via signals is one possible method for covering a large portion of the cosmos. These signals may have distinct frequencies making their identification difficult. The main focus of the SETI (Search for Extra-terrestrial Intelligence) programme so far has been on universal frequencies; a frequency in the range from 1420 MHz to 1720 MHz is known as *waterhole frequency* because water is thought to be necessary for life (at least as we know). It is a critical range for interstellar communication. Apart from the water hole frequency, Drake and Sagan suggested the frequency 56.8 GHz, i.e., the frequency associated with the cosmic microwave background, another frequency of universal nature, thus making it suitable for interstellar communication [60].

### 3.10 Temporal Explanations:

The SETI programme is only a little over six decades old. It is a fraction of a minuscule of cosmic time. An extra-terrestrial civilization could have existed and even perished long before the humans, or we must just wait and search.

### 3.11 Explanations Related to UFOs :

There have been numerous theories which associate alien life with Unidentified Flying Objects (UFOs) or Unidentified Aerial Phenomena (UAPs). These theories gained popularity in the mid-1940s. UFOs quickly became urban legends, with interesting stories such as Kenneth Arnold UFO sighting, The Roswell Incident, and The Belgian UFO Wave [61] getting popular. Claims about UFO sighting even went on to assert that governments and other bodies are either already in contact with alien civilizations, or have recovered the alien artefacts and are deliberately hiding them from the masses. Such conspiracy theories are often suspected to be part of pseudo-science; they provide little or no scientific evidence for their claims. Area 51, a United States Air Force installation in Nevada, not far from the Death Valley in Eastern California, is also mentioned in these theories as a secret base designed to contact extraterrestrial intelligence. Most of the hearsays concerning this place seem to be false. After all, it is only one of the several military grounds utilized for aircraft testing [62]. Recently, NASA has released a statement that it will conduct a study on the UAP (Unidentified Aerial Phenomenon) based on the existing and future data related to the UAP [63]. It is interesting that a well-established and responsible scientific agency is associating itself with UAP.

### 3.12 Our Limitations:

We must not forget our limitations. The first of these is our limited ability to detect extra-terrestrial intelligence. The SETI programme has only limited resources. With no idea where to look for and with detectors having limited

sensitivity, the limited hours of observation time only exacerbate the situation.

## 4. Conclusion

Both pessimists and optimists may have overestimated the strengths of their arguments. The solution to the Fermi paradox is possibly in our limitations. Human civilization has not made the lookout for extraterrestrial intelligence a priority. Government funding can only be limited, if at all any, since other needs of the society's needs are urgent. The SETI programme started with Project Ozma; it may appear to have failed but it has directly or indirectly inspired and motivated a whole generation of astronomers and biologists address the Fermi Paradox. Many projects followed the Project Ozma, including Project SERENDIP (Search for Extra-terrestrial Radio Emissions from Nearby Developed Intelligent Populations) in 1979, META II in 1990, Project BETA (Billion-channel Extra-Terrestrial Array) in 1995, Project Phoenix from February 1995 to March 2004, Project ATA (Allen Telescope Array) in 2007, and SERENDIP V in 2009.

When the Russian billionaire Yuri Milner founded the 'Breakthrough Initiatives' programme in 2015 [64], it reignited public interest in the SETI mission. *Breakthrough Listen*, *Breakthrough Message*, *Breakthrough Starshot*, *Breakthrough Watch*, and *Breakthrough Enceladus* are five initiatives in this programme. It is projected to last at least for ten years. The *Breakthrough Listen* is a \$100 million programme aimed at finding artificial extra-terrestrial signals by looking over 1,000,000 stars through dedicated watching hours at some of the world's finest observatories. A narrowband signal with characteristics roughly compatible with a technosignature approximately at 982 MHz was detected from Proxima Centauri. It first appeared to be an extra-terrestrial signal, subsequently found to be an earth-based artefact [65]. Jonathan Jiang and his team have created an

interstellar message which they plan to send soon into the cosmic ocean [66]. This will be the second attempt by the human civilization to send a message to an unknown civilization in the cosmos, after the famous "Arecibo Message," which was sent in 1973. Possibility of employing quantum communication is also being explored [53]. Further, in a recent discovery, scientists studying the environment of the exoplanet K2-18 b with the James Webb Space Telescope recently found a high level of the chemical dimethyl sulfide or some similar compound in its atmosphere. As all the detectable dimethyl sulfide in Earth's atmosphere is produced by microbes, it is often considered as a biosignature. But it will be too early to jump to conclusions, as it has been found that under Earth-like laboratory simulations dimethyl sulfide can be produced through non-living organisms as well, making it a non-reliable candidate for biosignature. Its tough measurement also leaves the possibility of its detection being an instrument artefact [70].

Although there is a long way to go, it seems hard to be pessimistic. NASA's ATLAS telescope

recently identified "3I/ATLAS," a comet-like interstellar object from outside our solar system that is scheduled to approach the sun around October 29, travelling close to Venus, Mars, and Jupiter on its way. While most astronomers consider it a comet, physicist Avi Loeb believes it may have been sent by an extraterrestrial civilization rather than a natural phenomenon. In support of the idea, he suggested that because of its retrograde orbital plane, the chances of it travelling close to the sun near Earth are only 0.2% if it is regarded a random event. Its huge diameter of 12 miles adds to his concerns. Although his views did not resonate with many in his own community, events like these raise this discourse time and again [71]. The rays of hope emerge from the recent advancements in computing powers enabled AI and ML systems that have the capacity to process substantially huge signals. This would facilitate various scenario analysis and simulations. In addition, AI and ML driven self-replicating space probes that are trained to understand extra-terrestrial signals and ability to transform may pave the way for setting newer horizons in understanding the extra-terrestrial intelligence.

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# Necessary Conditions for the Extensivity of Entropy

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## Abstract

The resolution of the Gibbs paradox is traditionally attributed to the inclusion of the Gibbs Correction Factor (GCF), which involves dividing the number of microstates or the partition function by  $N!$  to account for the indistinguishability of classical particles. While this combinatorial adjustment ensures the extensivity of entropy in many treatments, it is not sufficient in isolation. In this study, we argue that genuine extensivity of entropy in classical gases demands not only the GCF but also the fulfillment of the Classical Statistical Mechanics Condition (CSMC),  $n\lambda^3 \ll 1$ . This condition, which guarantees the suppression of quantum statistical effects, is essential to uphold the validity of classical approximations. By systematically analyzing this dual requirement, we present a more complete framework for understanding entropy in classical systems and offer a refined perspective on the resolution of the Gibbs paradox.

## 1 Introduction

Consider a gas of  $N$  non relativistic classical particles each with energy  $E = \frac{p^2}{2m}$ . In micro canonical ensemble, the number of micro states for classical particles is [1]

$$\Omega = \frac{\left(\frac{V}{h^3}\right)^N (2\pi m E)^{\frac{3N}{2}}}{\left(\frac{3N}{2}\right)!} \quad (1)$$

where  $V$  is the volume,  $h$  is Planck's constant,  $m$  is the mass of the particle and  $N$  is the number of particles. Substituting  $E = \frac{3}{2}NkT$ , which represents the thermal energy of a classical ideal gas (where  $k$  is the Boltzmann constant and  $T$  is the absolute temperature), and then taking the logarithm on both sides while applying Stirling's approximation, we obtain:

$$\ln \Omega \simeq N \ln \left( \frac{V(2\pi mkT)^{\frac{3}{2}}}{h^3} \right) + \frac{3N}{2} \quad (2)$$

Taking de Broglie wavelength  $\lambda = \frac{h}{\sqrt{2\pi mkT}}$  and using Boltzmann relation  $S = k \ln \Omega$ , entropy [2]

$$S = k \ln \Omega \simeq Nk \ln \frac{V}{\lambda^3} + \frac{3}{2}kN \quad (3)$$

Entropy is an extensive thermodynamic quantity, but the given equation is not extensive. To restore extensivity, Gibbs, in an ad hoc manner, divided  $\Omega$  by  $N!$ , leading to the extensive form of entropy [3, 4, 5, 6, 7, 8, 9]

$$S = k \ln \Omega \simeq Nk \left( \ln \frac{V/N}{\lambda^3} \right) + \frac{5Nk}{2} \quad (4)$$

This equation is assumed to be extensive based on the conclusion that the quantity within the bracket is non extensive, so that the equation becomes

$$S = N \times \text{Constant}$$

at constant temperature and volume. This means that  $S$  is having a linear dependence on the variable  $N$ .

## 2 Influence of the term $\ln \frac{V/N}{\lambda^3}$ on extensivity

$\frac{V}{N}$  is the volume available for a particle to occupy in the given system. Let

$$V/N = l^3$$

Using this Equation (5) becomes

$$S = Nk \ln \frac{l^3}{\lambda^3} + \frac{5Nk}{2} \quad (5)$$

We took different values of  $\frac{l^3}{\lambda^3}$  and plotted  $\ln \Omega$  vs  $N$  graph. The maximum value of  $N$

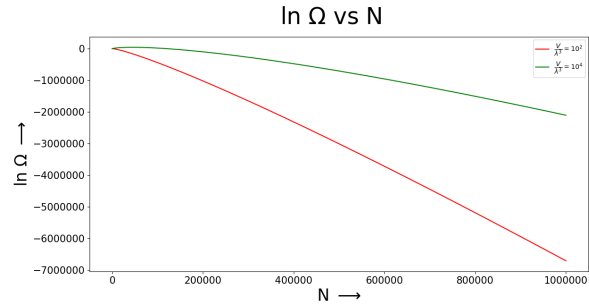


Figure 1:  $\ln \Omega$  vs  $N$  for  $\frac{l^3}{\lambda^3} = 10^2 \& 10^4$

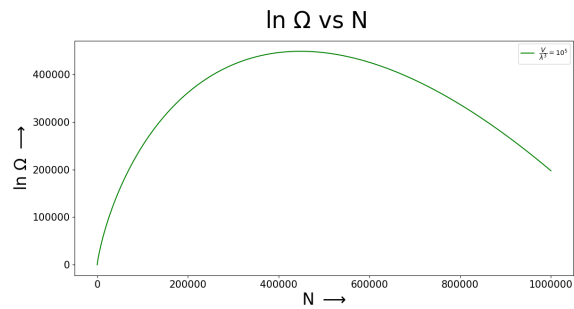


Figure 2:  $\ln \Omega$  vs  $N$  for  $\frac{l^3}{\lambda^3} = 10^5$

is taken as  $10^6$ . In Figure 1 and Figure 2 we had taken  $\frac{l^3}{\lambda^3} = 10^2, 10^4$  and  $10^5$ . We see that when  $\frac{l^3}{\lambda^3} \ll N$  the graph will not be linear which means we cannot get a constant entropy or extensive entropy which depends only on  $N$ . In Figure 3 we took  $\frac{l^3}{\lambda^3} = 10^8$  to  $10^{18}$  for  $N = 10^6$  and we got a linear graph. Thus entropy is only dependent on  $N$  which means extensive only when  $l^3 \gg \lambda^3$  which is the CSMC. This is not a surprising result because we always say that GCF is used for classical ideal systems( which will be obeying CSM condition).

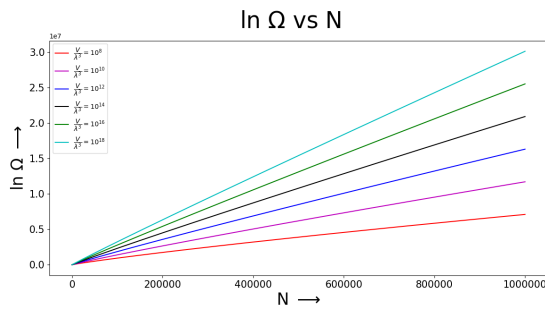


Figure 3:  $\ln \Omega$  vs  $N$  for  $\frac{l^3}{\lambda^3} = 10^8$  to  $10^{18}$

### 3 Conclusion

This work reinforces that the standard combinatorial resolution of the Gibbs paradox—dividing the number of microstates by  $N!$ —though essential, does not by itself guarantee the extensivity of entropy in classical systems. We have demonstrated that this resolution is valid only when the Classical Statistical Mechanics Condition (CSMC),  $n\lambda^3 \ll 1$ , is simultaneously satisfied. This condition ensures that quantum effects remain negligible and that the system can be meaningfully described by classical statistics. Importantly, this requirement is often left implicit or unacknowledged in traditional expositions. Our analysis clarifies that the proper reconciliation of thermodynamic and statistical mechanical entropy requires both the consideration of indistinguishability and the enforcement of the dilute limit. This dual perspective not only strengthens the conceptual basis of classical statistical mechanics but also sharpens its pedagogical value.

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